Scientific Note:  
\( xF_3^{\gamma Z} \) in charged lepton scattering

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Abstract. Measurements of the structure function \( xF_3^{\gamma Z} \) are now becoming available in charged lepton deep inelastic scattering at HERA. The correction factors which are necessary to compare these with measurements of \( xF_3^N \) in neutrino-nucleon scattering are deduced and a comparison is made between the H1 and the CCFR data. A sum rule is derived for the structure function \( xF_3^{\gamma Z} \) measured in charged lepton scattering which is the analogue of the Gross Llewellyn-Smith sum rule for neutrino-nucleon scattering.

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1 Introduction

At large squared four-momentum transfer, \( Q^2 \), the effects of \( Z^0 \) and \( W^\pm \) exchange become significant in charged lepton deep inelastic scattering relative to the single photon exchange process which dominates at lower values of \( Q^2 \). At such large values of \( Q^2 \), following the notation in [1], the cross section for unpolarised neutral current deep inelastic scattering, \( e^+p \rightarrow e^+X \), after correction for QED radiative effects is related to the structure functions of the proton by

\[
\frac{d^2 \sigma^{\pm NC}_{NC}}{dx^2 Q^2} = \frac{2\pi\alpha^2}{xQ^4} \left[ (1 + \delta_{NC}^{weak})(Y_+ \tilde{F}_2 + Y_- x \tilde{F}_3 - y^2 \tilde{F}_L) \right].
\]

(1)

Here \( \alpha \) is the fine structure constant, \( \delta_{NC}^{weak} \) are the weak radiative corrections described in [2], the helicity dependences of the electroweak interactions are contained in the functions \( Y_{\pm} = 1 \pm (1 - y)^2 \) and \( x \) and \( y \) are the Bjorken variables. The generalised structure functions \( \tilde{F}_2 \) and \( x\tilde{F}_3 \) can be decomposed as follows

\[
\tilde{F}_2 = F_2 - v_e \frac{\kappa_w Q^2}{Q^2 + M_Z^2} F_2^{\gamma Z} + (v_e^2 + a_e^2)(\frac{\kappa_w Q^2}{Q^2 + M_Z^2})^2 F_2^{Z} 
\]

(2)

\[
x\tilde{F}_3 = - a_e \frac{\kappa_w Q^2}{Q^2 + M_Z^2} xF_3^{\gamma Z} + (2v_e a_e)(\frac{\kappa_w Q^2}{Q^2 + M_Z^2})^2 xF_3^{Z},
\]

(3)

where \( \kappa_w^{-1} = 4M_Z^2/M_W^2 (1 - M_H^2/M_W^2) \) in the on-shell scheme [3] where \( M_W \) is defined in terms of the electroweak inputs. Here \( M_Z \) and \( M_W \) are the masses

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of the $W^\pm$ and $Z^0$ vector bosons. The quantities $v_e$ and $a_e$ are the vector and axial vector couplings of the electron to the $Z^0$ and are related to the weak isospin of the electron, namely $v_e = -1/2 + 2 \sin^2 \theta_w$ and $a_e = -1/2$ [3] where $\theta_w$ is the electroweak mixing angle. The electromagnetic structure function $F_2$ originates from photon exchange only. The functions $F_2^\gamma$ and $x F_3^\gamma$ are the contributions to $F_2$ and $x F_3$ from $Z^0$ exchange and the functions $F_2^{\gamma Z}$ and $F_3^{\gamma Z}$ are the contributions from the interference of the $\gamma$ and $Z^0$ exchange amplitudes. The longitudinal structure function $x F_L$ may be similarly decomposed.

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The structure function $x F_3$ can be measured in charged lepton deep inelastic scattering by taking the difference between the measured cross sections for electron and positron scattering, as described in [1]. In this way the major contributions from $\tilde{F}_2$ and $\tilde{F}_L$ are eliminated. In the kinematic range of HERA the contribution of $xF_3$ is small and can be safely neglected.

In the quark parton model of the nucleon the structure functions $xF_3^\gamma$ and $xF_3^\gamma$ are related to the electroweak couplings and an incoherent sum of the parton distribution functions (PDFs)

$$[xF_3^\gamma, xF_3^\gamma] = x \sum_q [2e_q A_q, 2v_q A_q] (q - \bar{q})$$

(4)

where $q, \bar{q}$ are the quark PDFs, $e_q$ is the charge of the quark, $v_q = I_3q - 2e_q \sin^2 \theta_w$ and $A_q = I_3q$ are the quarks’ electroweak couplings with $I_3q$ the weak isospin of the quark. In neutrino nucleon scattering, averaging over the scattering from neutron and proton targets and assuming from isospin conservation that the valence $u$ quarks in the neutron have the same distribution as the valence $d$ quarks in the proton and vice versa, $xF_3^N$ has a similar relation, of the form

$$xF_3^N(x, Q^2) = \sum_q x(q(x, Q^2) - \bar{q}(x, Q^2)).$$

(5)

2 Comparison of charged lepton and neutrino data

It can be seen from equations (4) and (5) that the measured value of $xF_3^N$ in $\nu$-Nucleon and $xF_3^\gamma$ in charged lepton scattering are comparable within the Quark-Parton model after allowance is made for the different targets and electroweak couplings. Both are related to the differences of the PDFs for quarks and antiquarks which are dominated by the valence quarks of the target nucleon. Taking the ratio of equations (4) and (5), substituting the $u$ and $d$ quark charges and couplings, gives

$$\frac{xF_3^\gamma(x, Q^2)}{xF_3^N(x, Q^2)} = \frac{2}{3} \left( \frac{1 - \frac{1}{2} d_u}{1 + \frac{1}{2} u_d} \right)$$

(6)

where the quark-antiquark differences have been replaced by the valence distributions, $u_v$ and $d_v$.

Hence it is possible to compare the $\nu$ and charged lepton data by correcting one of them by the factor on the right hand side of equation (6). The correction

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Fig. 1. The H1 data on $x F_3^\gamma Z$ compared with the CCFR data at $Q^2 = 12.6$ GeV$^2$ multiplied by the ratio given in equation (6). This ratio was computed from the values of $u_v$ and $d_v$ taken from the QCD fit described in [5]. The smooth curves (labelled H1 97 PDF Fit) are also calculated from this fit.

The factor for the quark distributions is calculable and is not very sensitive to the difference between the $u_v$ and $d_v$ distributions. For example, if one makes the naive assumption that $d_v/u_v$ is 1/2 then

$$\frac{1 + \frac{1}{2} \frac{d_v}{u_v}}{1 + \frac{d_v}{u_v}} \sim 0.83.$$  

(7)

Taking the values of $d_v$ and $u_v$ from the latest parton distribution functions [4] this correction factor varies from 0.811 at the lowest $x$ to 0.921 at the highest $x$ values. If instead we use the fit to the H1 data [5] the factor varies from 0.826 to 0.923 over the same range of $x$. Furthermore, the $Q^2$ dependence of this factor is very weak since there is a cancellation from the ratio $d_v/u_v$. Hence there is 1-2% uncertainty coming from the quark distribution functions in evaluating the correction factor to compare neutrino data on $x F_3^\gamma Z$ and charged lepton data on $x F_3^{\nu N}$.

Figure 1 shows the H1 measurement [1] of $x F_3^\gamma Z$ after introducing the small correction for the scaling violations determined from the H1 97 PDF Fit [5]. Weighted means have been taken of the measurements corrected to $Q^2 = 1500$ GeV$^2$ at the same $x$ value. The smooth curves show the expected variation from the fit to fixed target and H1 data. The H1 data are compared to the CCFR measurements of $x F_3^{\nu N}$ [6] after correction by the ratio computed from eq. (6).

The difference between the curves at $Q^2 = 12.6$ GeV$^2$ and $Q^2 = 1500$ GeV$^2$ indicates the softening of the structure function expected from the DGLAP evolution for a non-singlet structure function. The evolution of such a structure function is very safe since it does not depend on the gluon distribution of the target nucleon. In contrast the evolution of a singlet structure function is less.
safe because of its dependence on this gluon distribution.

3 A sum rule for $xF_3^\gamma\gamma$ in charged lepton scattering

The structure function $F_3^\gamma\gamma$ in charged lepton scattering should obey the sum rule

$$\int_0^1 F_3^\gamma\gamma \, dx = (2e_u a_u N_u + 2e_d a_d N_d) \left( 1 - \mathcal{O}(\frac{\alpha_s}{\pi}) \right) = \frac{5}{3} \left( 1 - \mathcal{O}(\frac{\alpha_s}{\pi}) \right).$$ (8)

This follows from replacing the difference between the quark distributions in equation (4) by the valence quark distributions. The fact that there are $N_u=2$ $u$ and $N_d=1$ $d$ valence quarks in the proton allows the integrals over these distributions to be replaced by $N_u$ and $N_d$. This gives the prediction in equation (8) to which QCD radiative corrections $\mathcal{O}(\alpha_s/\pi) \approx 5\%$ should be applied [7]. A similar sum rule (the Gross Llewellyn-Smith sum rule [8]) has been observed to be valid for neutrino data [9]. Evaluating the integral over the H1 data gives

$$\int_{02}^{65} F_3^\gamma\gamma \, dx = 1.88 \pm 0.35(\text{stat.}) \pm 0.27(\text{sys.}).$$ (9)

The value expected from the standard model (obtained by integrating over the solid curve in Fig. 1) is $\int_{02}^{65} F_3^\gamma\gamma \, dx = 1.11$ and over the whole range is $\int_0^1 F_3^\gamma\gamma \, dx = 1.59$. Here the difference of the latter from $5/3$ arises from the QCD radiative corrections. The H1 measurement is 1.7 standard deviations above the value expected from the standard model.

It should be noted that the sum rule is an integral over $F_3^\gamma\gamma$ and the values of the integrand are derived from measurements of $xF_3^\gamma\gamma$. Hence there is considerable uncertainty in the measured integral coming from the data at low $x$ because of the need to divide the measurement in each bin by $x$.

4 Discussion of the result

Figure[11] indicates that the H1 data on $xF_3^\gamma\gamma$ have a tendency to become softer than expected from pure DGLAP evolution for a non-singlet structure function. The sum rule integral described in the previous section shows that the effect is significant at roughly the level of 1.7 standard deviations and it could be within the experimental error. The future high luminosity running at HERA is needed to study the $x$ dependence of $xF_3^\gamma\gamma$ in greater detail.

The radiative corrections were examined closely to see if they could contribute a systematic effect. A difference between $e^+$ and $e^-$ scattering [10] is expected from the interference of two photon exchange with the single photon exchange diagram. Such corrections are implemented in the HERACLES generator but not in the DJANGO Monte Carlo used to correct the data [11]. Studies of the HERACLES generator showed that this effect tends to increase all the values of $xF_3^\gamma\gamma$ derived from the data by a magnitude which is small compared to the size of the errors. Hence it is valid to neglect such corrections in the present data. It may become necessary to include these corrections as the data become more precise following the luminosity upgrade of HERA.

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5 Conclusion

The new measurements by H1 of $x F_3^{\gamma Z}$ in charged lepton neutral current deep inelastic scattering at HERA have been discussed and compared to the neutrino data on $x F_3^{\nu N}$. A new sum rule is described which governs the integral of $F_3^{\gamma Z}$ and the H1 data are compatible with this sum rule within 1.7 standard deviations. Hence, although the H1 data tend to be systematically above the prediction of the standard model, the difference from the model is within the experimental precision. High precision measurements of $x F_3^{\gamma Z}$ will become possible with the increased luminosity soon to be available at HERA, allowing this structure function to be studied in greater detail.

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