Lecture 2

- Quantum Particles
- Experimental Signatures
- The Exchange Model
- Feynman Diagrams

Eram Rizvi

Royal Institution - London
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Outline

A Century of Particle Scattering 1911 - 2011
- scales and units
- overview of periodic table \(\rightarrow\) atomic theory
- Rutherford scattering \(\rightarrow\) birth of particle physics
- quantum mechanics - a quick overview
- particle physics and the Big Bang

A Particle Physicist's World - The Exchange Model
- quantum particles
- particle detectors
- the exchange model
- Feynman diagrams

The Standard Model of Particle Physics - I
- quantum numbers
- spin statistics
- symmetries and conservation principles
- the weak interaction
- particle accelerators

The Standard Model of Particle Physics - II
- perturbation theory & gauge theory
- QCD and QED successes of the SM
- neutrino sector of the SM

Beyond the Standard Model
- where the SM fails
- the Higgs boson
- the hierarchy problem
- supersymmetry

The Energy Frontier
- large extra dimensions
- selected new results
- future experiments
If a ‘particle’ has an associated wave - where is it?
If particle has a single definite momentum it is represented by a single sine wave with fixed $\lambda$
But - wave is spread out in space - cannot be localised to a single point

Particle with less well defined energy: i.e. a very very narrow range of momentum $\Delta p$
$\Rightarrow$ several sine waves are used to describe it
They interfere to produce a more localised wave packet confined to a region $\Delta x$
The particle’s position is known better at the expense of knowing its momentum!

This is the origin of the Heisenberg Uncertainty Principle

$$\Delta p \Delta x > h$$

The quantum world is fuzzy!
Cannot know precisely the position and momentum
The trade-off is set by Planck’s constant $h$
h is small $\Rightarrow$ quantum effects limited to sub-atomic world

$$h = 4.135 \times 10^{-15} \text{ eVs}$$
Wave functions and Operators

Macroscopic objects also have associated wave functions etc
But wavelength is immeasurably small!

\[ \lambda = \frac{h}{p} \]

How is information about the particle ‘encoded’ in the wave function?
The wave function describes and contains all properties of the particle - denoted \( \psi \)
All measurable quantities are represented by a mathematical “operator” acting on the wave function

A travelling wave moving in space and time with definite momentum (fixed wavelength/frequency) can be written as:

\[ \psi = A \sin \left( \frac{2\pi x}{\lambda} - \omega t \right) \]

A = amplitude of the wave
\( \omega = \) frequency
\( \lambda = \) wavelength

We choose a position in space, \( x \), and a time \( t \) and calculate the value of the wave function

Can also write this in the form:

\[ \psi = Ae^{i(kx-\omega t)} \quad \text{and} \quad k = \frac{2\pi}{\lambda} \]

(ignore \( i \) for now)

If this represents the wave function of particle of definite (fixed) energy \( E \) then a measurement of energy should give us the answer \( E \)
We now posit that all measurements are represented by an operator acting on the wave function. Which mathematical operation will yield the answer $E$ for the particle energy?

\[ i\hbar \frac{\partial}{\partial t} \]

This is the derivative with respect to time, equal to the rate of change of something. A derivative calculates the slope of a mathematical function. This is incomplete - it needs something to act on, just like $+$ is incomplete without $x$ and $y$ to act on (i.e., $x+y$). It acts on the wave function $\psi$.

Like a verb without a noun to act on.

An object's velocity is the rate of change of its distance.

\[ \text{slope} = \frac{\text{(change in distance)}}{\text{(change in time)}} \]

An object's acceleration is the rate of change of its velocity.

\[ \text{slope} = \frac{\text{(change in velocity)}}{\text{(change in time)}} \]
Wave functions and Operators

For a particle with wave function and definite energy $E$ then:

$$i\hbar \frac{\partial}{\partial t} \psi = E \psi$$

This wave notation makes derivatives easier to calculate

$$\psi = Ae^{i(kx-\omega t)}$$

Similarly measurement of momentum for a particle with definite momentum $p_x$ has the operator equation:

$$\frac{\hbar}{i} \frac{\partial}{\partial x} \psi = p_x \psi$$

In both cases the operator leaves the wave function unchanged
It is just multiplied by the momentum, or energy

(mathematically $E$ and $p$ are the eigenvalues of the equation)
Imagine a free particle moving in space - no forces acting on it
Kinetic energy = total energy for the particle

classical equation \[ E = \frac{p^2}{2m} \]
but in quantum mechanics \[ p \rightarrow \frac{\hbar}{i} \frac{\partial}{\partial x} \]

operator for total kinetic energy
(kinetic energy is energy due to motion)
m = particle mass

For a free particle moving in 1 dimension with no forces acting on it and with definite energy:

1d Schrödinger equation
co-ordinate position x \[
-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi = i\hbar \frac{\partial}{\partial t} \psi
\]

Notice:
derivatives with respect to spatial co-ordinates are related to momenta
derivatives with respect to time co-ordinate is related to energy
The Schrödinger Equation

In three dimensions (co-ordinate positions x, y, z):

\[
-\hbar^2 \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi = i\hbar \frac{\partial}{\partial t} \psi \quad \rightarrow \quad \frac{-\hbar^2}{2m} \nabla^2 \psi = i\hbar \frac{\partial}{\partial t} \psi
\]

Finally we include an interaction of the particle with an external (potential) energy field V

\[
\frac{-\hbar^2}{2m} \nabla^2 \psi + V(x, y, z) \psi = i\hbar \frac{\partial}{\partial t} \psi
\]

\(\psi\) contains all info about our particle
Equation describes how wave function changes momentum and energy when interacting with an energy field V

Rule for Quantum Mechanics:
Use classical equations of physics
Replace all “observables” with quantum operators acting on the particle wave function
Observables = energy, momentum, angular momentum - anything we can measure

(we have neglected relativity though - beyond our scope here)
We will return to this later...
Scattering Experiments

To measure the structure size $x$ use wavelengths of similar size - the probing scale

Don't use a finger to probe the structure of a sand grain!

Shorter wavelengths = higher energy $\Rightarrow$ need more energetic colliders!
The ATLAS experiment at the LHC

The Atlas Experiment
7000 tonnes
Mass of the Eiffel Tower
Half the size of Notre Dame
Data rate: 20,000,000 Gb/s

The Atlas Collaboration
3500 physicists
174 universities
38 countries
Measuring cross-section of a process requires recognising event properties:

- Electromagnetic energy with a charged track: e+ or e-
- Electromagnetic energy without track: photon
- Collimated ‘jet’ of particles: gluon/quark induced jet
- Penetrating charged track: μ+ or μ-
- Missing transverse energy: ν
- Missing longitudinal energy: beam remnants
- Displaced secondary vertex: in-flight decay of 'long lived' particle

Look at the event topology...
Large experiments needed to measure outgoing particles from collisions
Experiment consists of layered detectors each sensitive to different types of particle
Look for signatures of particle types
LHC Collision Events

2 jets of particles: quarks / gluons
two penetrating particles
opposite charge
Electromagnetic energy with associated particle track
Missing energy in transverse direction
What's going on in these collisions?
We measure reaction rates!
Rate at which particles are produced versus energy, momentum, angle...
Called a cross section
related to the probability of a specific reaction occurring

\[ n + p \rightarrow n + p \]

Like Rutherford scattering
Expect largest reaction rate for small angle scattering

Why is reaction rate equally large here???
Equally likely to have head-on collision as glancing collision??

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**Reaction Rate**

- Low energy proton / neutron scattering

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**Diagram:**

- Plot of reaction rate vs. proton scattering angle for different energies.

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**Legend:**

- 91 MeV
- 105 MeV
- 137 MeV
- 153 MeV
- 200 MeV
- 290 MeV
- 580 MeV
- 590 MeV
Neutron - Proton Scattering

large angle scattering

This reaction should occur far more infrequently!

particles exchange identity!

The interaction must:
- exchange momentum
- exchange electric charge

something happens here
changes proton ↔ neutron
An exchange particle is forbidden 
violates energy-momentum conservation

Imagine you are moving alongside the proton at equal speed 
As far as you can measure the proton is at rest compared to you 
Where does it get the energy to emit a particle from?

Saved by the Heisenberg Uncertainty Principle:

\[ \Delta E \Delta t > h \]

Small energy \( \Delta E \) can be ‘borrowed’ for a time \( \Delta t = h / \Delta E \)!

This process is an interaction - it is the expression of a force of nature 
Newton: force = rate of change of momentum (F=ma)

What can we predict about the exchange particle?

\( \Delta E \) is ‘used’ to produce the particle with mass - what is it? 
Nuclear force acts between n and p - has a range of 1 fm 
Assume it travels at light speed \( c \) - how long does it live for? 
\[ c \Delta t = 1 \text{ fm} \quad & \quad \Delta E = mc^2 \]

\[ mc^2 \approx \frac{hc}{c \Delta t} \quad \text{So m} = 200 \text{ MeV/c}^2 \]

1/5\(^{th}\) of proton mass 
400 times electron mass

Our bizarre model predicts new particle:  
- mass \( \sim 200 \text{ MeV} \)  
- charge = +1 
- responsible for force of attraction inside nucleus
Predicted by Hideki Yukawa in 1934
Also showed that electrostatic force can be described by a massless particle particle exchange
Both particles have spin = 0 or 1
⇒ the photon!

1947 - three particles discovered: the pion

<table>
<thead>
<tr>
<th>particle</th>
<th>charge</th>
<th>mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0$</td>
<td>0</td>
<td>135.0</td>
</tr>
<tr>
<td>$\pi^+$</td>
<td>+1</td>
<td>139.6</td>
</tr>
<tr>
<td>$\pi^-$</td>
<td>-1</td>
<td>139.6</td>
</tr>
</tbody>
</table>

The exchange model seems to work!
Extra two particles describe interactions of nn and pp scattering
Modern quantum field theory: fundamental interactions visualised as Feynman diagrams

- **Space**
  - A
  - B
  - C
  - D
  - X

- **Time**

**Real World**

- A
  - B
  - C
  - D
  - X

**Quarks and Leptons**

**Photons, W and Z**

**Gluons**

**Particles**

**Antiparticles**
Feynman diagrams have an intuitive simplicity
They are more than just aids to visualisation
They represent precise mathematical equations
They allow quantitative predictions for reaction rates!

Examples of Feynman diagrams

Particle exchange is the manifestation of a force!

$q\bar{q} \rightarrow e^+ e^-$

$e q \rightarrow e q$

$e^+ d \rightarrow e^+ d$
A ‘di-jet’ event at high energy

or:

or:
Decay of a long-lived composite particle

- Two oppositely curved tracks
- Penetrating tracks
- Displaced secondary vertex
Production and decay of a W boson particle (carrier of the weak force)
Higher energy → probing particle interactions further back in time millionths of a second after the big bang

Forces of nature start to behave in similar ways
Consider them as manifestations of a single unified high energy force