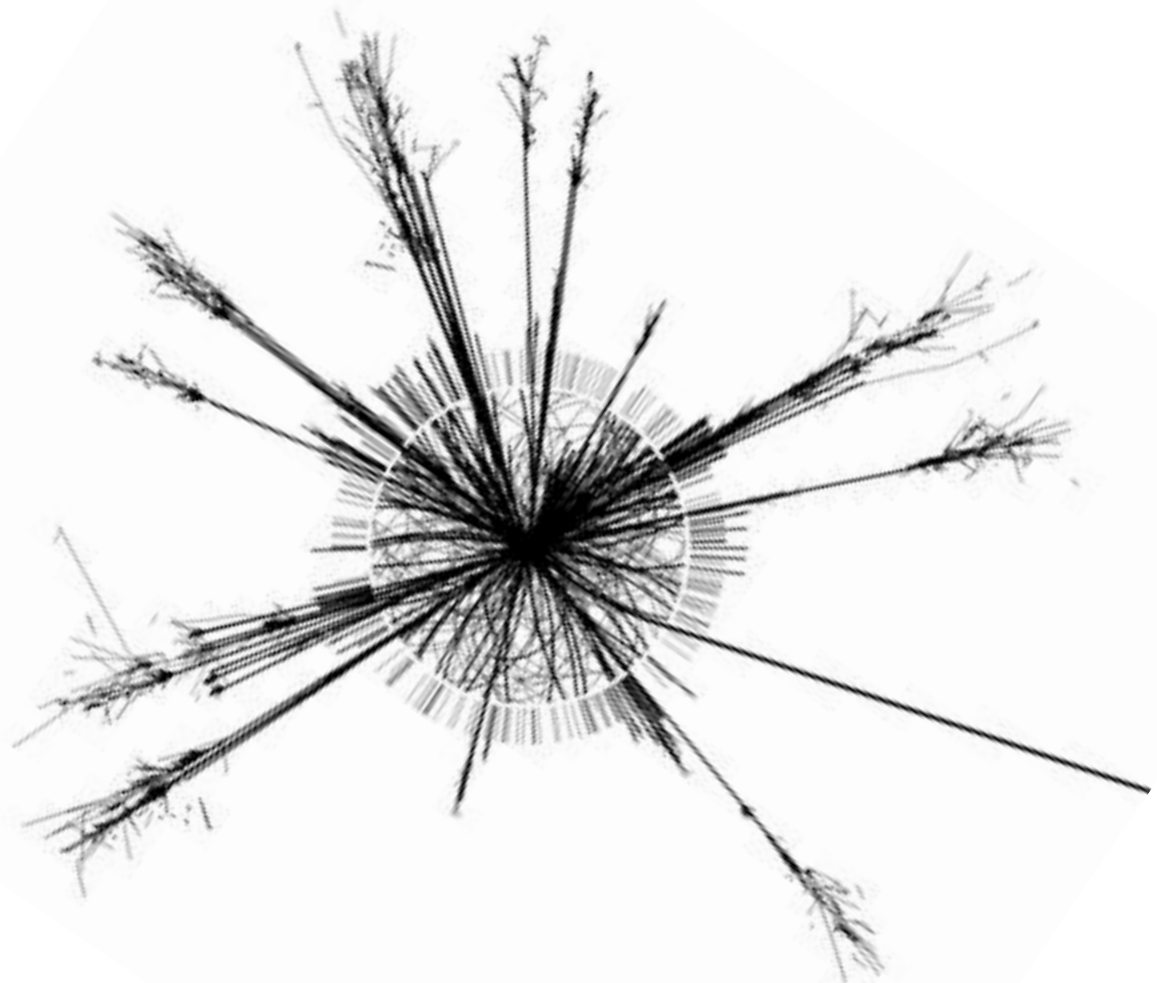
Quantum Mechanics and Particle Scattering



Lecture 1

- Introduction to the Course
- Scales and Units
- Rutherford Scattering - 1911
- The New Physics - Quantum Mechanics

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7th February 2012

Queen Mary University of London

Who Am I ?

Always wanted to be a physicist
Heard about discovery of two new particles the W and Z in 1981 (BBC2 Horizon)
Wanted to be a particle physicist ever since
I now work with some people featured in that programme

Studied Physics at Manchester University
Graduated 1st class in 1993
PhD in Particle Physics from Queen Mary, London
Joined new collider experiment: HERA - high energy and unique electron-proton accelerator in Hamburg
Awarded PhD in 1997
Research Fellow at HERA laboratory - measured quark structure
Postdoc with Birmingham University - precision proton structure measurements
Lecturer at Queen Mary, London - teaching Nuclear Physics, Scientific Measurement, undergraduate tutorials
Tutor for undergraduate admissions to Physics
Postgraduate admissions tutor for Particle Physics research group

Research focus in 3 areas:
- leading team of researchers finalising measurements of proton structure from HERA (2 months to go!)
- joined Atlas experiment on LHC - co-ordinating a measurement of quark/gluon dynamics
- author and project leader of team producing state-of-the-art simulations for micro-black holes at LHC
- starting involvement to design a 'trigger' system for an upgrade to the LHC in 2018

These are my dream jobs!

Course Objectives

- describe the Standard Model in terms of the fundamental interactions between the quarks and leptons
- have a qualitative understanding of Feynman diagrams and relate these to experimental measurements
- understand the connection between conservation principles and symmetries
- describe the observation of neutrino mixing / neutrino velocity
- understand the successes and limitations of the Standard Model
- describe how some of these limitations are overcome in alternative models
- understand the aims of current experiments including the LHC and T2K
- understand the results of Higgs boson searches

Will use simple mathematics to motivate some arguments

Will go step-wise in explaining equations e.g. Schrödinger equation:

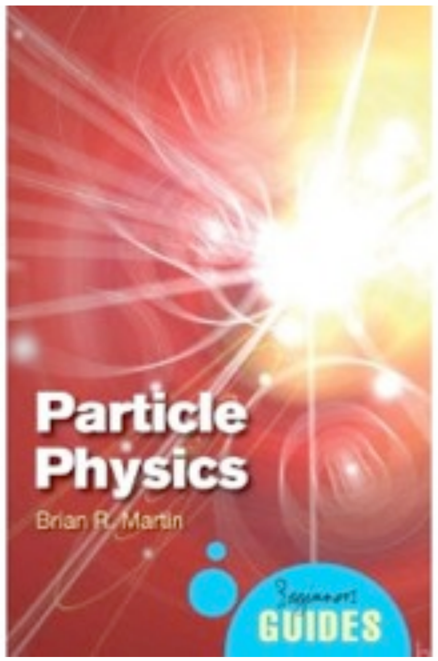
$$-\frac{\hbar^2}{2m}\nabla^2\Psi(r) + V(r)\cdot\Psi(r) = E\cdot\Psi(r)$$

I've made some assumptions about who you are!
• broadly aware of scientific developments
• perhaps with a science degree
• read new scientist / scientific american type magazines?
• scientists in a different field
• interested amateurs

Some or all of this may be wrong!
Difficult for me to know what level to pitch at
Tell me if its too hard / too simple
Feel free to email me complaints / suggestions

Recommended Books

No books cover the material as I would like
Often too basic or too detailed



Brian R. Martin
Amazon: £7
Paperback: 216 pages
Publisher: Oneworld Publications (1 Mar 2011)
ISBN-10: 1851687866
ISBN-13: 978-1851687862

Several books by Frank Close - excellent author

A good background reference: hyperphysics website: http://hyperphysics.phy-astr.gsu.edu
Some figures taken from there and gratefully acknowledged
Some figures also taken from wikipedia

6 lectures - 90 mins each
7pm every thursday eve
No home works!

Outline

A Century of Particle Scattering 1911 - 2011

- scales and units
- overview of periodic table → atomic theory
- Rutherford scattering → birth of particle physics
- quantum mechanics - a quick overview
- particle physics and the Big Bang

A Particle Physicist's World - The Exchange Model

- quantum particles
- particle detectors
- the exchange model
- Feynman diagrams

The Standard Model of Particle Physics - I

- quantum numbers
- spin statistics
- symmetries and conservation principles
- the weak interaction
- particle accelerators

The Standard Model of Particle Physics - II

- perturbation theory & gauge theory
- QCD and QED successes of the SM
- neutrino sector of the SM

Beyond the Standard Model

- where the SM fails
- the Higgs boson
- the hierarchy problem
- supersymmetry

The Energy Frontier

- large extra dimensions
- selected new results
- future experiments

"In the matter of physics, the first lessons should contain nothing but what is experimental and interesting to see. A pretty experiment is in itself often more valuable than twenty formulae extracted from our minds."

- Albert Einstein

My approach will be experimentally driven

I believe that experiment is the final arbiter of the truth

Only experiment can decide between the validity of two competing models or theories

I will attempt to motivate statements with experimental data

This is the heart of scientific methodology

What is Particle Physics ?

Study of the phenomena of fundamental and sub-atomic particles

- To understand the structure of matter at the smallest distance scales
- Understand the details of their interactions in terms of fundamental forces
- To understand the relationships between the particles of the standard model
- To search for new particles and new interactions not yet observed
- To understand the origin of mass
- To attempt to incorporate gravity as a quantum force of nature
- To understand the matter / anti-matter asymmetry in the universe

The Standard Model of Particle Physics

Worlds most successful theory to date - Describes fundamental constituents of matter

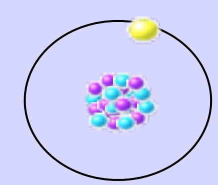
u d s c b t quarks: strong, weak, electromagnetic

e μ τ charged leptons: weak, electromagnetic

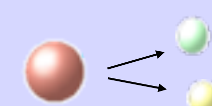
ν_e ν_μ ν_τ neutrinos: weak

gluons Strong: holds atomic nucleus together

photons Electromagnetic: binds atom together



W and Z bosons Weak: radioactive decay processes



No description of Gravity at sub-atomic level

Electromagnetic & Weak parts of Standard Model are known extremely precisely

Theory of strong interactions is less well understood

Timeline of Discoveries

1895 Discovery of X-rays by Wilhelm Röntgen

1896 Henri Becquerel discovers of radioactivity

1897 Thompson discovers the electron

1911 Discovery of the atomic nucleus by Rutherford

1913 Bohr model of atom

1914 Determination of nuclear charge

1919 Rutherford discovers the proton

1926 Quantum mechanics: Schrödinger equation is born

1931 Pauli predicts neutrino in beta decay

1932 Discovery of the neutron – Chadwick

1933 Discovery of positron - anti-matter

1934 Fermi develops theory of neutrino

1935 Yukawa:exchange model of particle interactions

1946 First meson discovered

1950 Quantum field theory of Electromagnetism

1955 Discovery of anti-proton

1956 Parity Violation in beta decay

1959 Discovery of the neutrino

1960s/70s Many sub-atomic particles discovered

1964 Discovery of Ω^- particle

1970s Quantum-chromodynamics & quarks

1970s Electroweak theory is proposed

1974 Discovery of charm quark

1975 Discovery of tau lepton

1978 Discovery of bottom quark

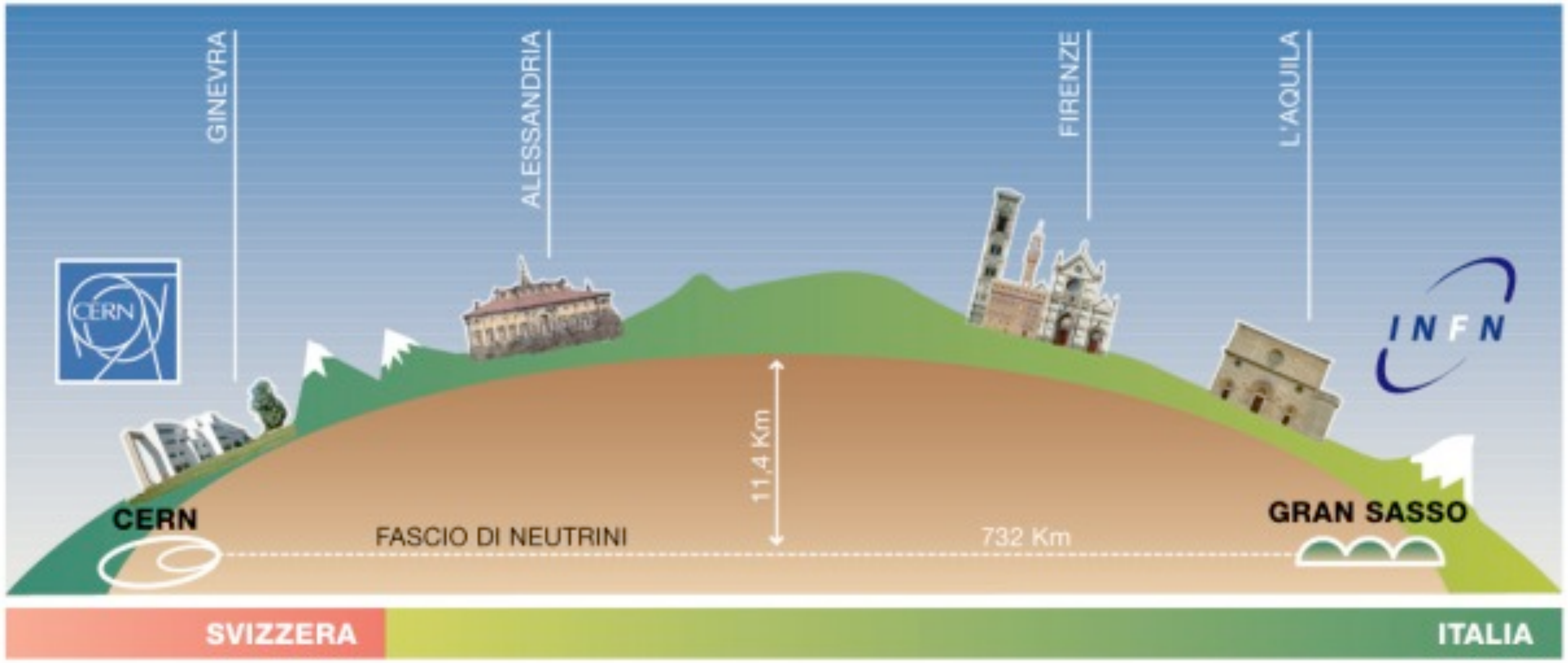
1979 Discovery of the gluon

1983 Electroweak theory experimentally verified

1995 Discovery of top quark

1998 Neutrino oscillations observed

2012 ??



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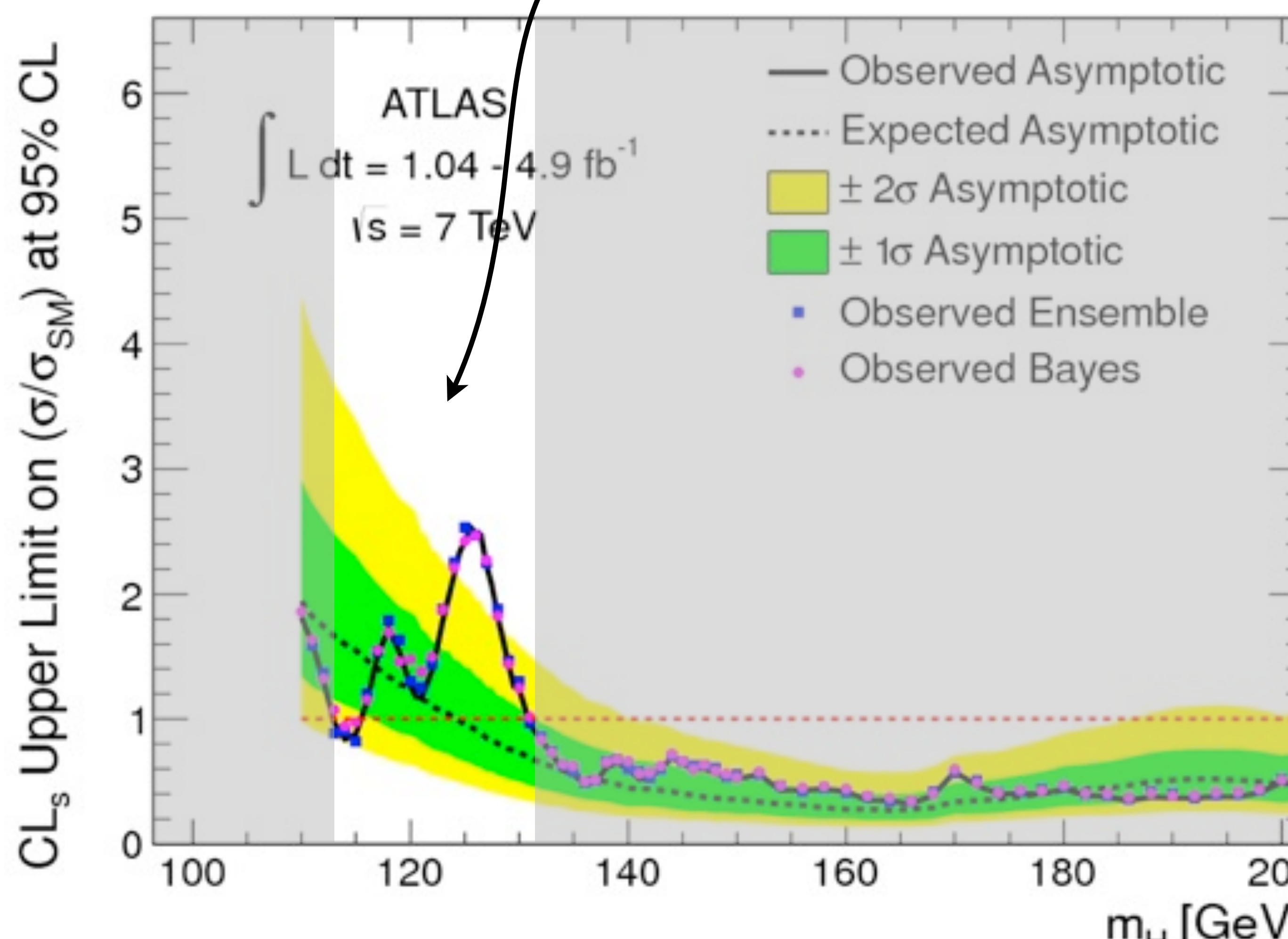
What velocity do neutrinos travel at?

??

Is the Higgs hiding here?



Nomenclature - The Boring Stuff

Other nomenclature:
particles often written as a symbol

α - alpha particle from nuclear decays
β - beta particle from radioactive decays, known to be an electron
ɣ - photon
p - proton
e - electron

composite particles often written with electric charge superscript: K^0
usually left off for fundamental particles unless distinction is required

anti-matter particles often denoted with a bar on top $\overline{p}$

some exceptions: anti-electron e^+

mesons - just distinguished by electric charge π^+, π^0, π^-

Scales - Typical Sizes and Energies

Powers of 10:

Range of numerical values covered in physics is large

Very cumbersome to write 0.00001 etc

This notation makes rough calculations easy:

Volume of a proton is $[1\times10^{-15}\,\mathrm{m}]^3 = 1\times10^{-45}\,\mathrm{m}^3$
To square or cube a number - multiply exponents 15 x 3 = 45

Mass of a proton = 1.67×10^{-27} Kg

What is the density?
Density = mass / volume
= $1.67\times10^{-27} / 1\times10^{-45}$
To divide numbers subtract the exponents
Density = $1.67 \times 10^{(-27 - (-45))}$
= 1.67×10^{18} Kg m^{-3}

Compare to density of water = 10^3 Kg m^{-3}
15 orders of magnitude difference
1000,000,000,000,000 times more dense than water

Difficult to visualise
Thinking in terms of a difference in time:
15 orders of mag is the same difference between 1s and 100 million years

number	notation	prefix	symbol
	10^{18}	exa-	E-
	10^{15}	peta-	P-
	10^{12}	tera-	T-
	10^{9}	giga-	G-
1000000	10^{6}	mega-	M-
1000	10^{3}	kilo-	K-
100	10^{2}		
10	10^{1}		
1	1		
0.1	10^{-1}		
0.01	10^{-2}		
0.001	10^{-3}	milli-	m-
0.000001	10^{-6}	micro-	μ-
	10^{-9}	nano-	n-
	10^{-12}	pico-	p-
	10^{-15}	femto-	f-
	10^{-18}	atto-	a-

Scales - Typical Sizes and Energies

In physics - use SI units:

distance:	metre
time:	second
mass:	kilogram
energy:	joule

For everyday objects and situations this works well

Handling subatomic particles is not an everyday occurrence!
SI units can be used in particle physics...
...but they are awkward

e.g. proton mass = 1.67×10^{-27} Kg

Use a new system of units specifically for this area of physics
We are free to choose any system of units **provided we are consistent**
Never mix units!!!

Units in Particle Physics

Distance – the fermi (fm)

1 fermi = 10^{-15} m = 1 fm
proton radius ~ 1 fm

Time – the second (s)

Our familiar unit of time measurement
Range of particle lifetimes varies enormously:
lifetimes ~10^{-12} s i.e. 1 picosecond up to millions of years (~10^{13} s)

Energy – the electron volt (eV)

The energy required to accelerate 1 electron through a 1V potential
1 eV = 1.602 x 10^{-19} J (conversion rate is electron charge in Coulombs)
Typical nuclear energies are in MeV range (10^{6})
Typical rest energies are much larger ~ GeV (10^{9}) more on this later...

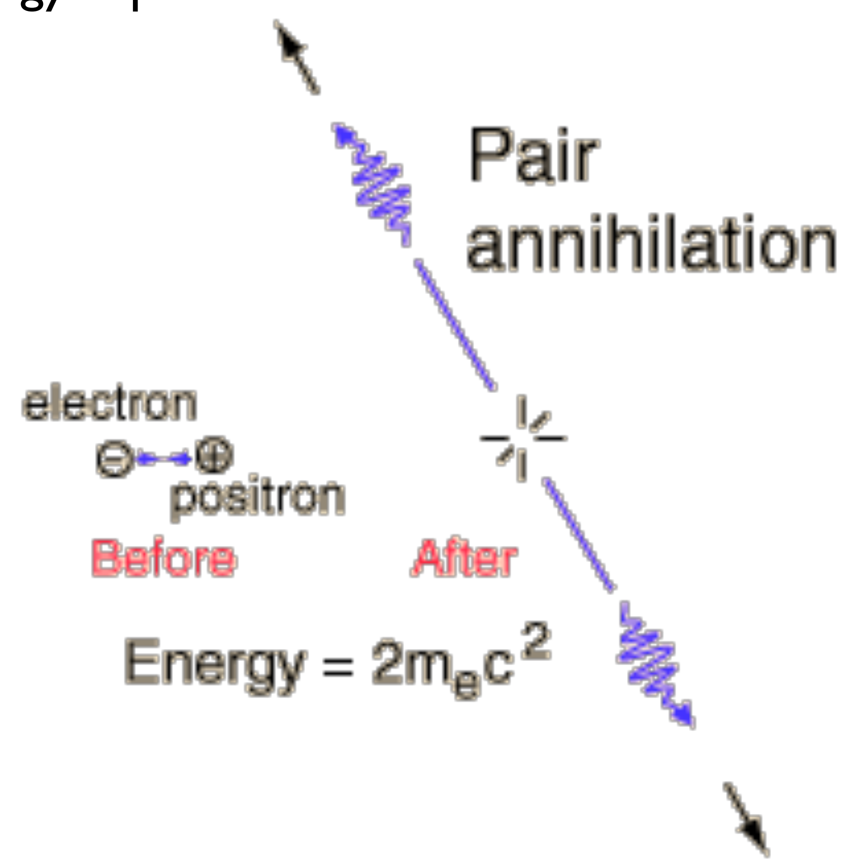
<u>Mass</u> – MeV/c^2

Since E=mc^2 we can switch between mass & energy as we please
Mass and energy are equivalent

€ and £ are equivalent currencies - exchange rate is 1.11407 €/£

Conversion rate between mass and energy is $c^2 = (2.99 \times 10^8\ ms^{-1})^2 = 8.94 \times 10^{16}\ m^2s^{-2}$!!
$\Rightarrow$ small amount of mass = large amount of energy

Use this to define units of mass i.e. the energy equivalent
Simplifies calculations:
If a electron and anti-electron collide and annihilate how much energy is produced?
electron mass = anti-electron mass = 0.511 MeV/c^2
$\Rightarrow$ energy produced = $(0.511\ MeV/c^2 + 0.511\ MeV/c^2)\ c^2$
$= 2 \times 0.511\ MeV/c^2 \times c^2 = 1.022\ MeV$

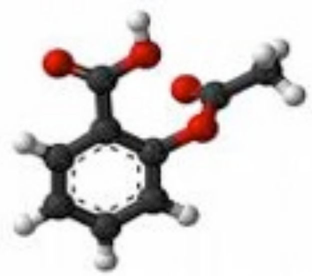


Never multiply any numerical result by 2.99 x10^8 ms^{-1}
If you do this, you are probably making a mistake!!!

Scales - Typical Sizes and Energies

Sizes
Everyday Matter ~1m

Typical Energies
0.01 eV - thermal energies



Molecule 10^{-9} m

1 eV - binding energy of molecule

Atom 10^{-10} m

10 eV – 1 KeV

Nucleus 10^{-14} m

1 MeV – 10 MeV

Proton 10^{-15} m

1 GeV

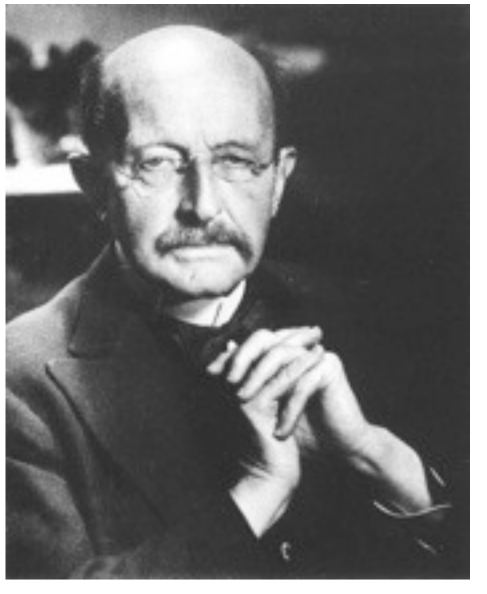
nucleus is 4 orders of magnitude smaller than atom

The World of Physics in 1911

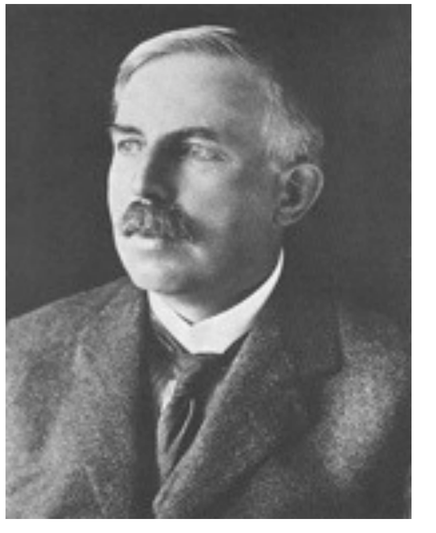
Over 100 years of discovery and experimentation

Discovery of electron	- Thompson 1897
Birth of quantum physics	- Planck 1900
Relativity	- Einstein 1905
Atomic structure	- Rutherford 1911

Thompson



Planck



Rutherford

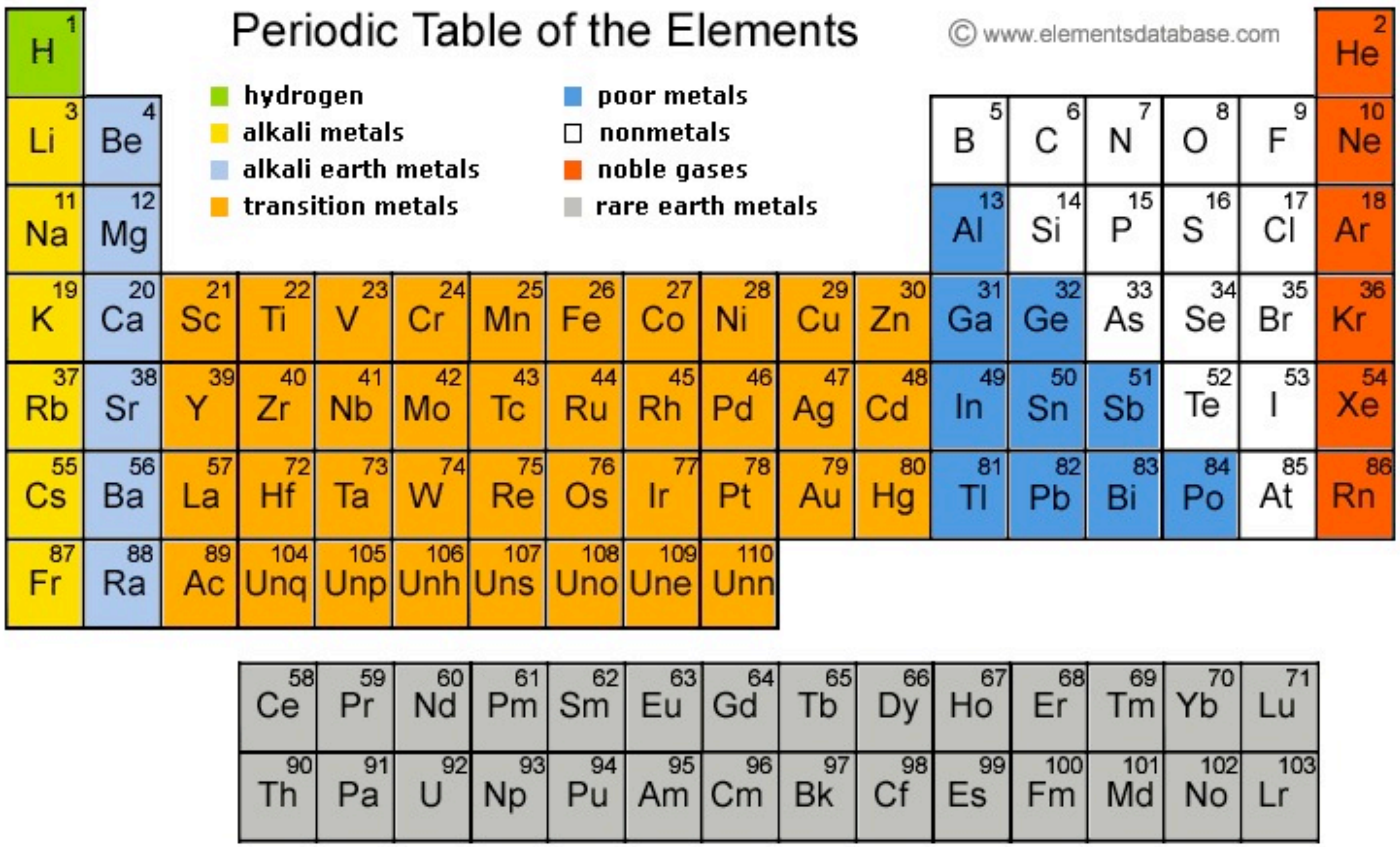
Structure of matter at the start of the 20th Century

Atoms organised into a table of elements

Arranged by their <u>chemical properties</u>

Experiments performed to determine atomic mass, and how they react

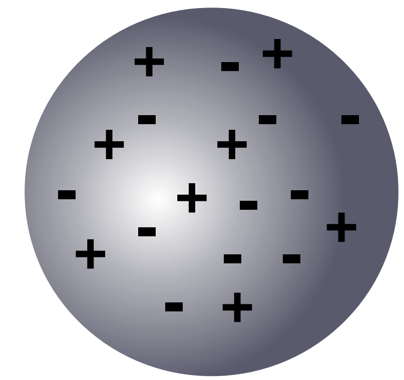
See what happens when we mix two of these together!



Rutherford Scattering

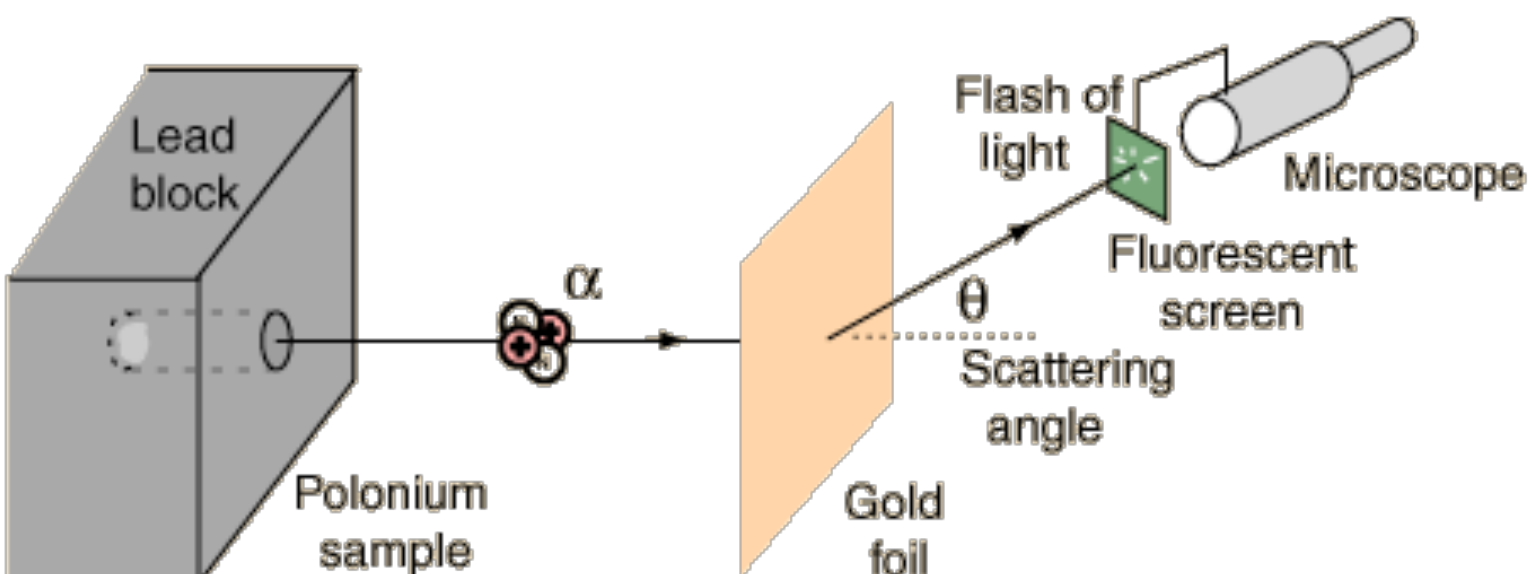
Rutherford's experiment was ground-breaking
First particle scattering experiment
Set the stage for next 100 years
Use a small subatomic particle to probe the structure of matter

The "plum pudding" model of the atom



In 1900 we knew atom was divisible
- neutral object containing electrons
- electrons embedded in a blob of positive matter??

Test this by firing a small projectile at target atom - observe how it scatters
Rutherford used an alpha particle (helium nucleus - 2 protons, 2 neutrons)
charge = 2+



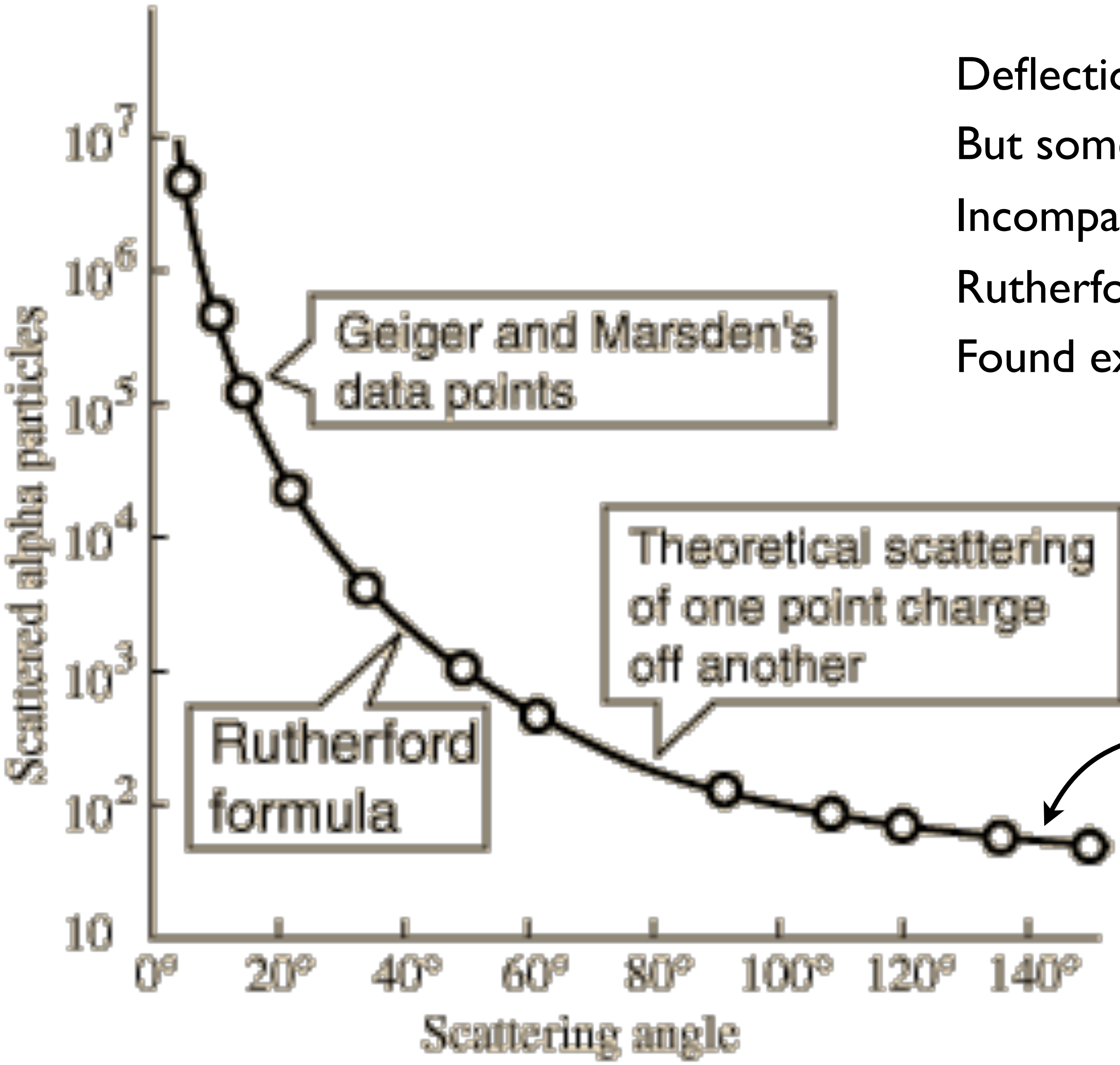
Scattering is due to electric charges

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

F = force
q_1 = charge on alpha particle
q_2 = small bit of charge in atom
r = separation distance

plum pudding model predicts small deviations less than 0.02°

Rutherford Scattering



Deflections due to multiple interactions - many random collisions

But sometimes very large deflections - rare!

Incompatible with the multiple scattering ⇒ single hard scatter

Rutherford proposed model of dense atomic nucleus

Found experiment described his model expectation

Reaction rate as function of scattering angle:

$$\frac{d\sigma}{d\Omega} = \left(\frac{Zze^2}{16\pi\epsilon_0 T} \right)^2 \frac{1}{\sin^4 \theta/2}$$

T = energy of α particle
Z = atomic number of target (79 for gold)
z = atomic number of probe particle (2 for α)
e = charge of electron
ε_0 = permittivity of free space - how readily the vacuum allows electric fields to propagate

- First evidence that atom consists of <u>very</u> dense small nucleus
- 99.95% of atomic mass is in nucleus
- Nuclear radius is 10,000 times smaller than atomic radius
- Remainder is "empty space"

First scattering experiment to elucidate structure!

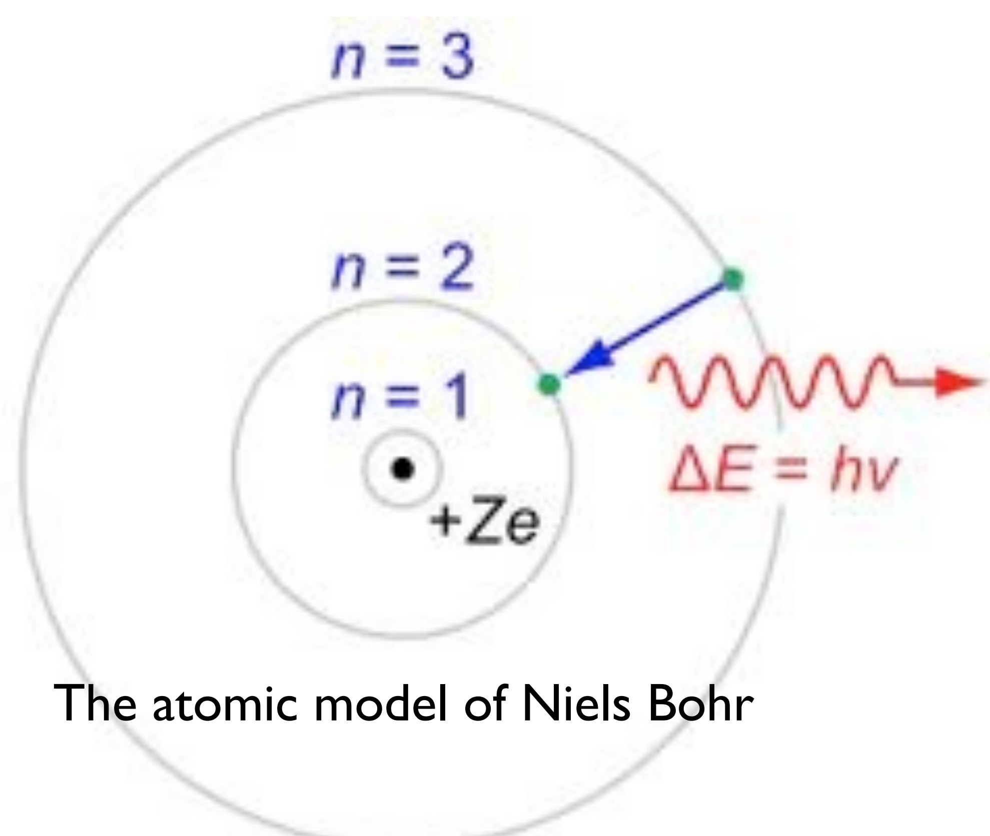
Early part of 20th century opened to way to quantum physics
Standard physics had several problems irreconcilable with experiment

Energy is quantised - comes in discreet packets
For light this depends only on frequency ω
Conversion factor is Planck's constant h = 6.6x10^{-34} J.s = 4.1x10^{-15} eV.s

$$E = h\omega$$

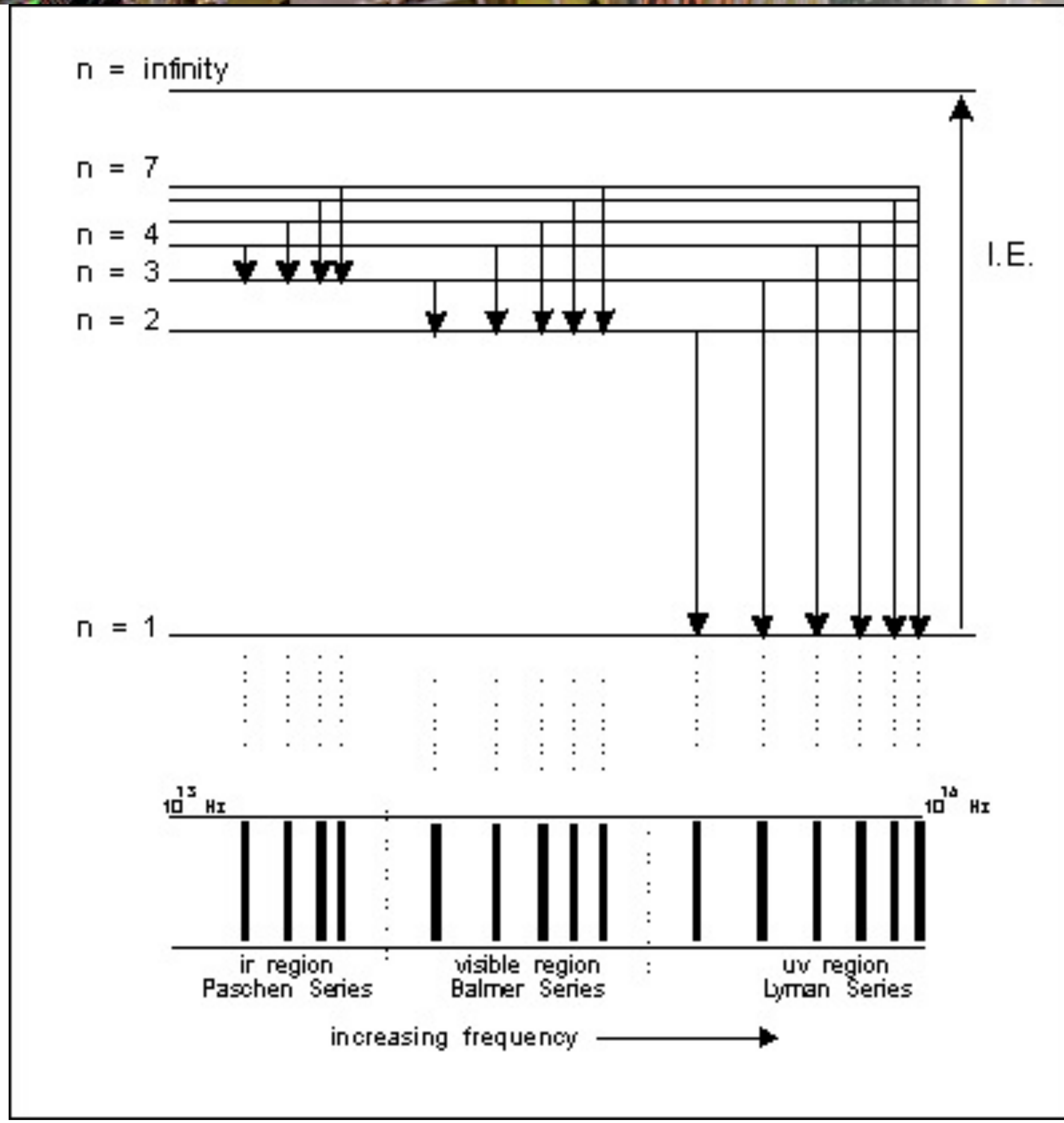
For a given frequency - quantum of energy is always the same
For 450 nm wavelength ⇒ 666 x 10^{12} Hz frequency ⇒ E = 2.75 eV always!

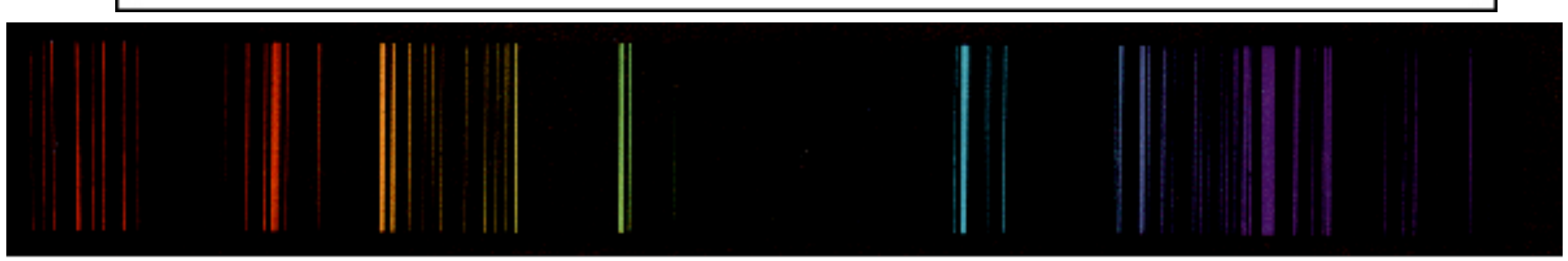
World without quantisation:
electrons orbiting atomic nucleus would radiate energy
⇒ spiral inwards - all atoms unstable!



The atomic model of Niels Bohr

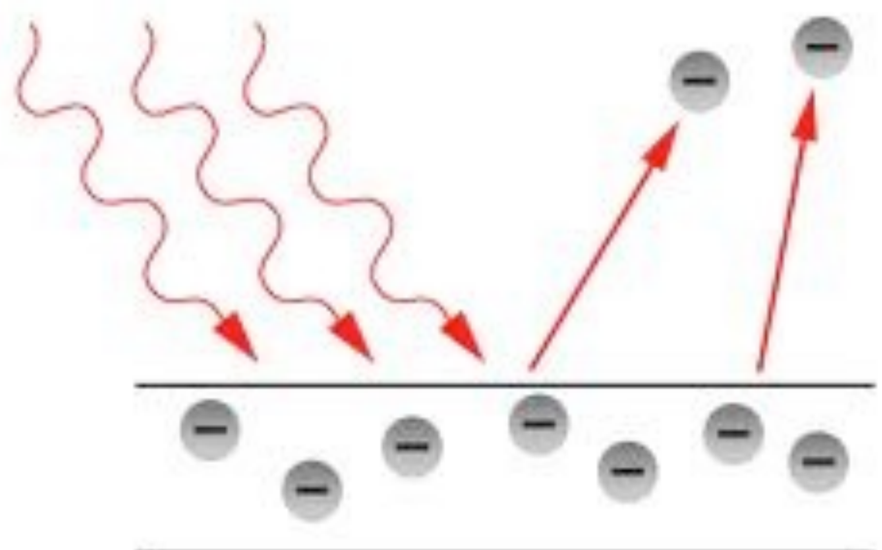
Spectral emissions lines:
the “fingerprint” of an atom





Increasing Frequency →

Photons can liberate electrons from a metal

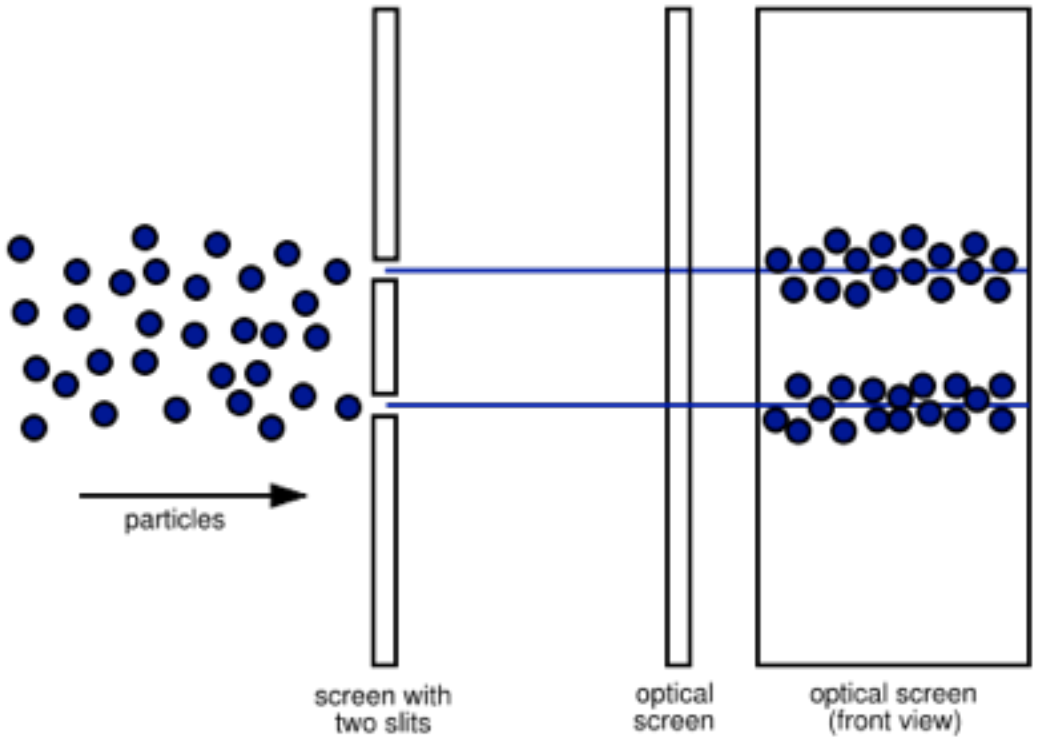


- No electrons liberated below a threshold frequency
- Energy of liberated electrons depends on frequency only
- Increasing intensity of radiation liberates more electrons

$$E = h\omega$$

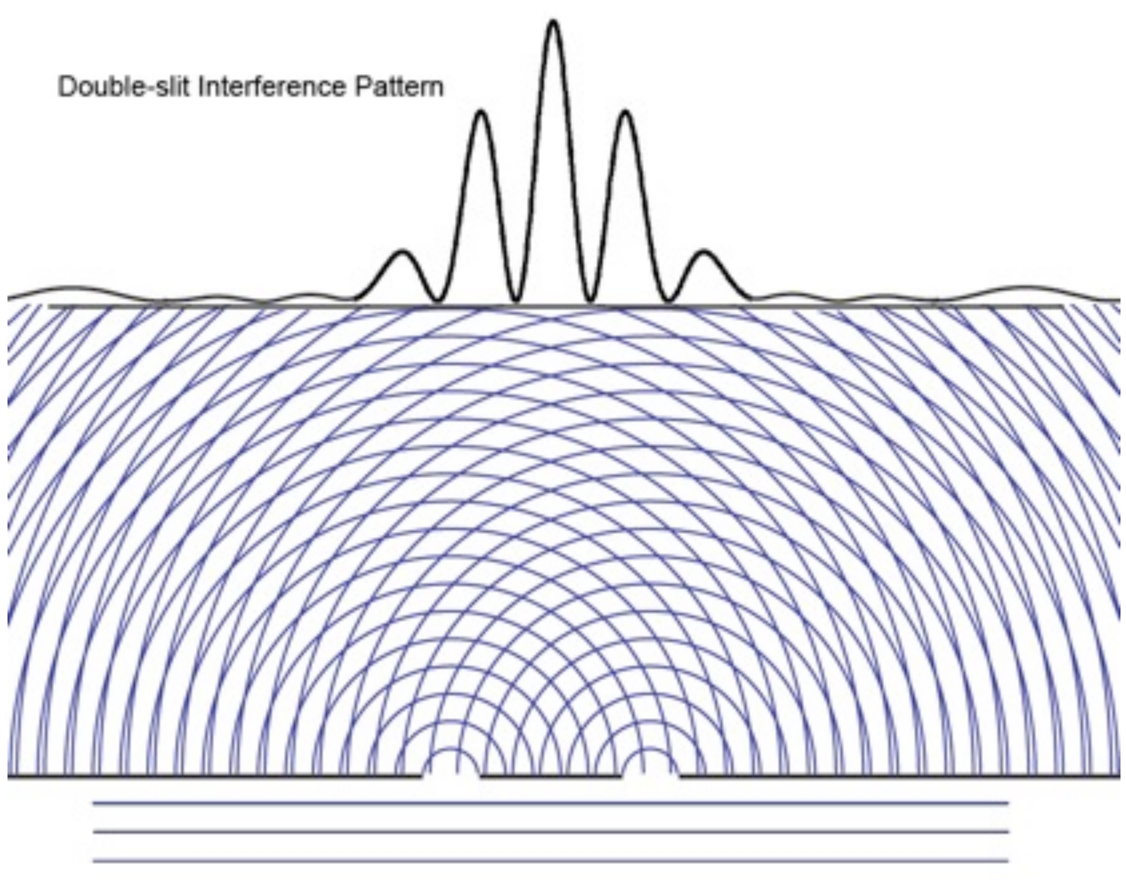
E = energy
ω = frequency

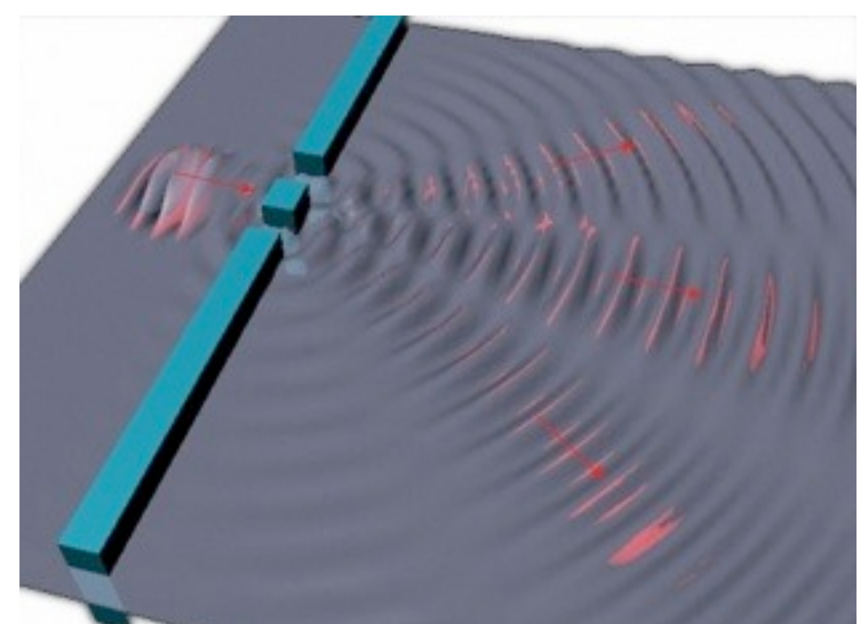
Wave Particle Duality



Classic double slit experiment
Particles fired at a screen with two slits
Record image of particles which pass through the slits
Two intensity bands are observed

repeat the experiment with waves:
several intensity bands are observed
due to wave interference at both slits



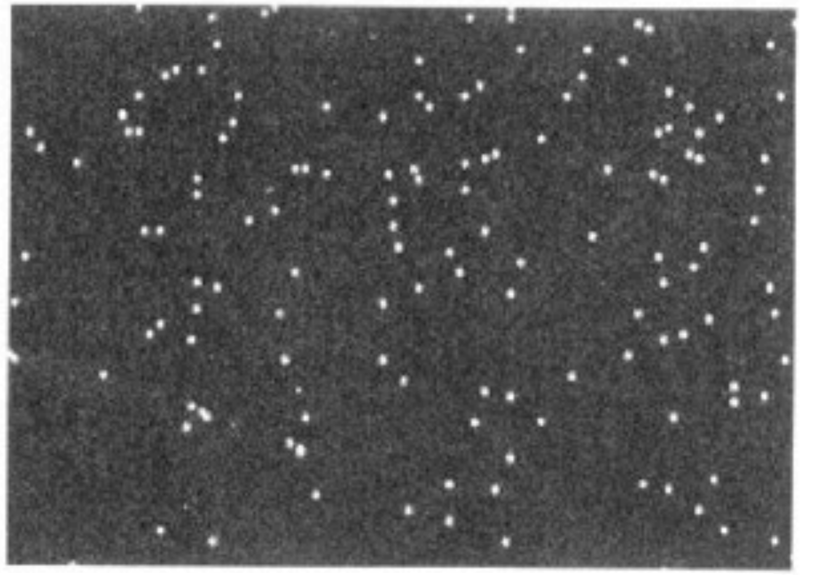


Perform experiment with particles
interference pattern is observed!

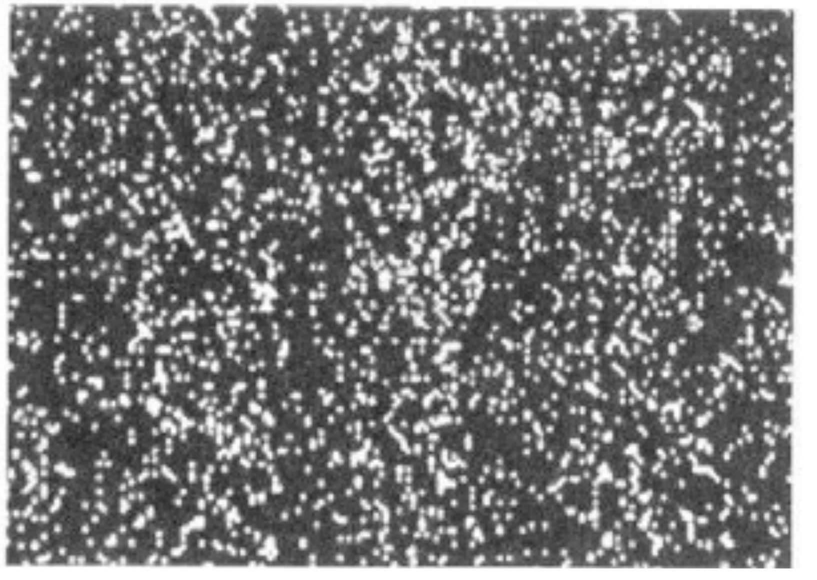
Repeat experiment - fire 1 particle at a time
observe intensity pattern build up
Still observe interference pattern!

Conclusion:
→ particles behave like waves
or
→ single particle enters both slits and interferes with itself

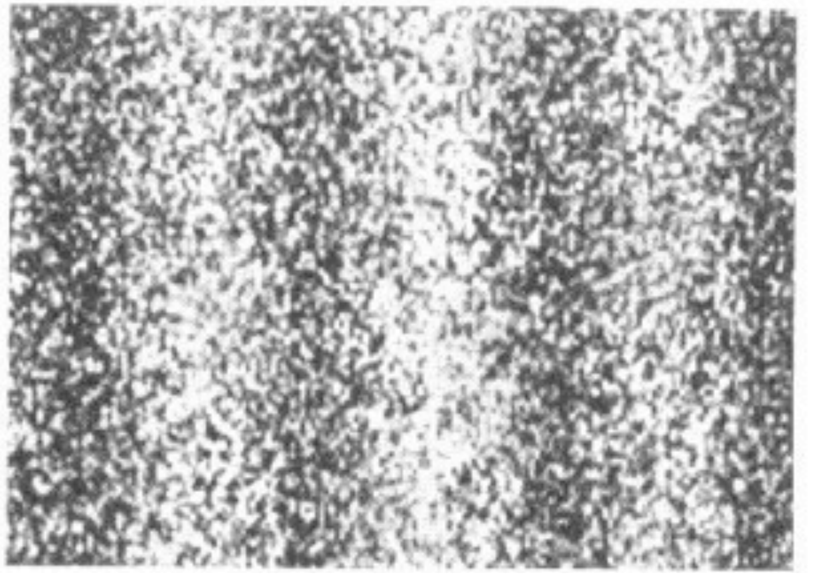
What if you place a detector near each slit
which slit did particle enter?
Interference pattern is destroyed!
Wave nature of matter is gone
The act of observation is part of the story



100 electrons in double slit experiment



3,000 electrons in double slit experiment



70,000 electrons in double slit experiment

Wave Particle Duality

What are these matter waves?

All particles have an associated frequency - Louis De Broglie 1924

Nothing is actually oscillating

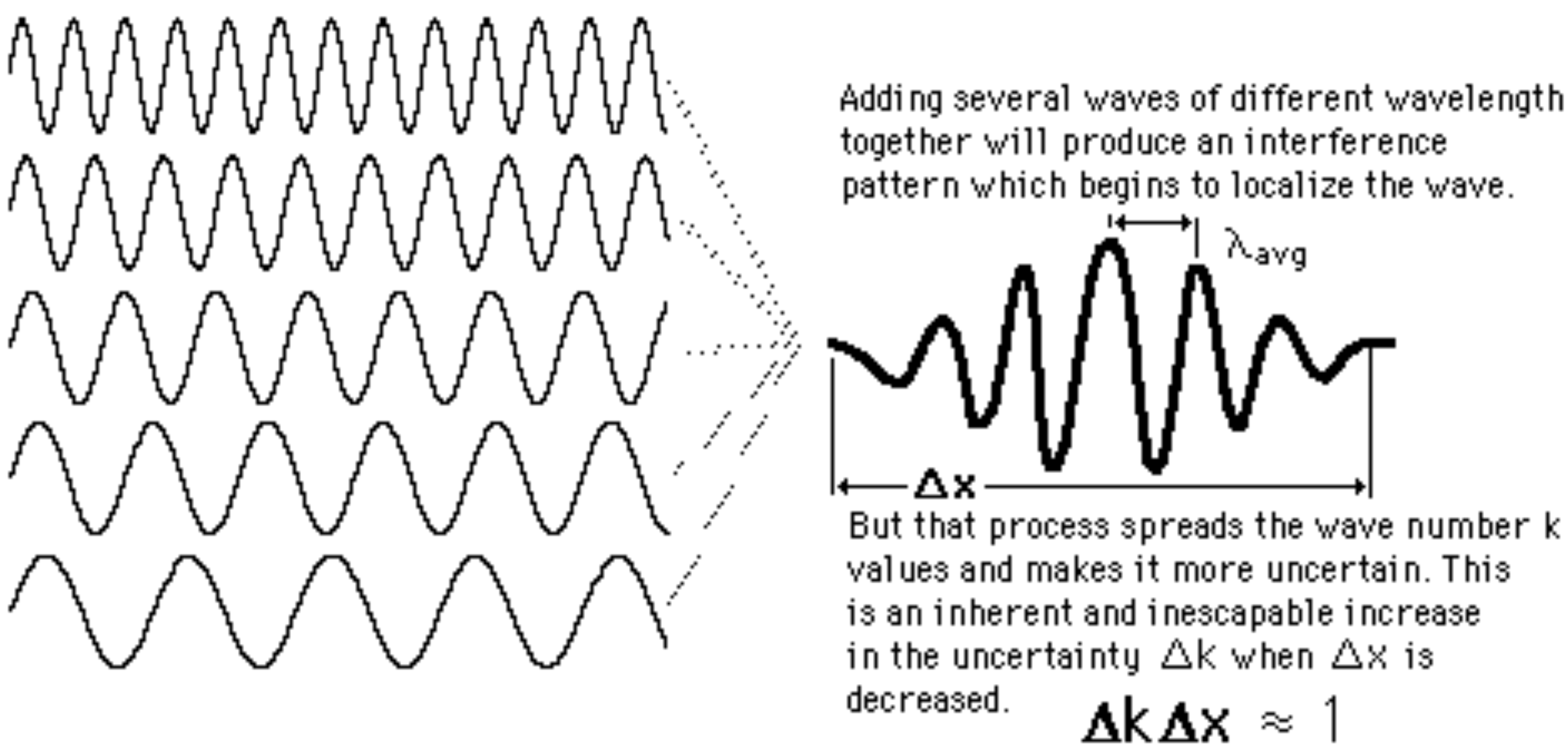
Cannot observe wave directly - only its consequences, e.g. interference

Oscillation frequency directly proportional to particle's energy (strictly momentum)

$$\lambda = \frac{h}{p}$$

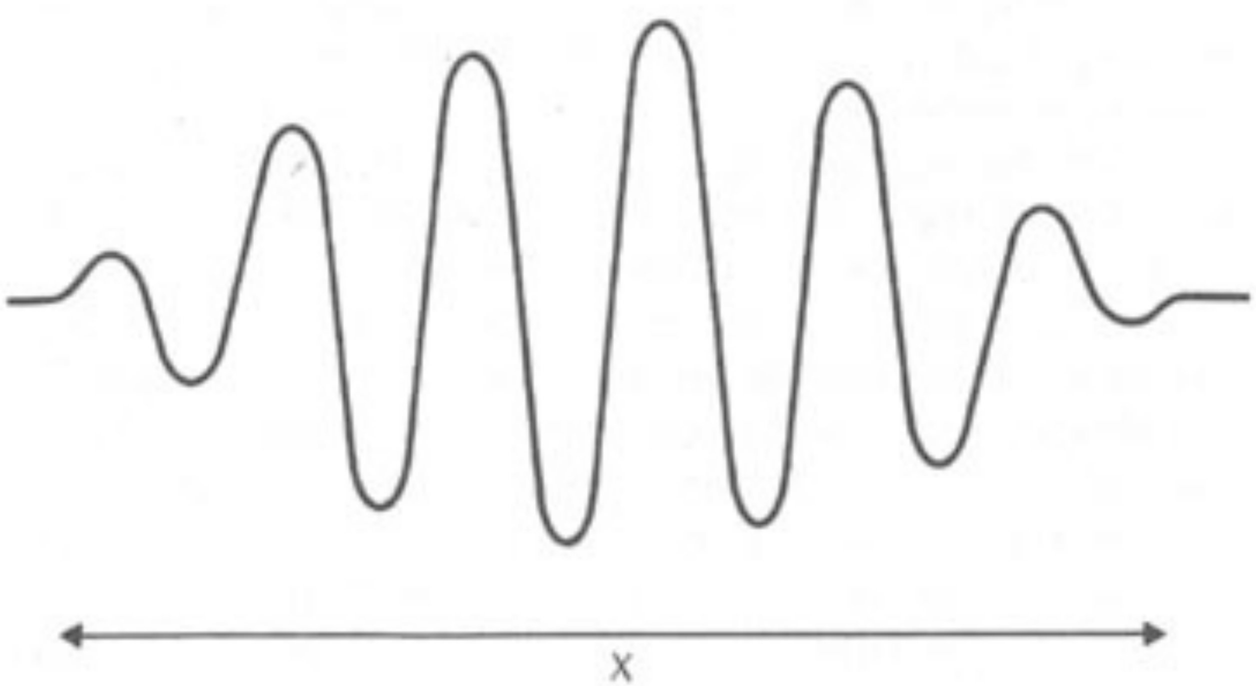
p = momentum

λ = wavelength



If a 'particle' has an associated wave - where is it?
If particle has a single definite momentum it is represented by a single sine wave with fixed λ
But - wave is spread out in space - cannot be localised to a single point

Particle with less well defined energy: i.e. a very very narrow range of momentum Δp
$\Rightarrow$ several sine waves are used to describe it
They interfere to produce a more localised wave packet confined to a region Δx
The particle's position is known better at the expense of knowing its momentum!



a wave packet corresponding to a particle located somewhere in the region X

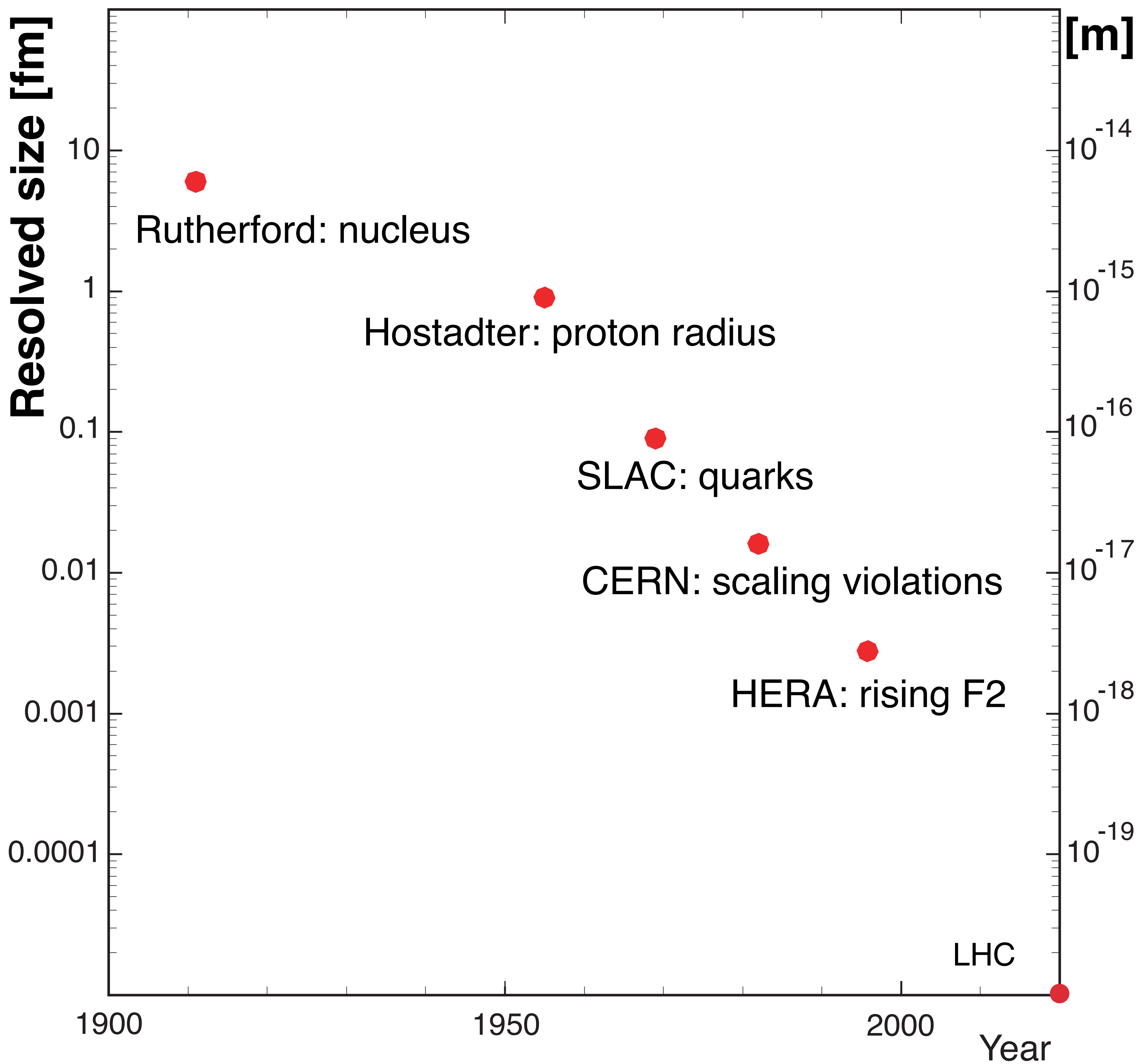
This is the origin of the Heisenberg Uncertainty Principle

$$\Delta p \Delta x > h$$

The quantum world is fuzzy!
Cannot know precisely the position and momentum
The trade-off is set by Planck's constant *h*
h is small $\Rightarrow$ quantum effects limited to sub-atomic world

The Last 100 years

logarithmic scale: 6 orders of magnitude!



publication date of experiment

$$\lambda \simeq \frac{hc}{E}$$

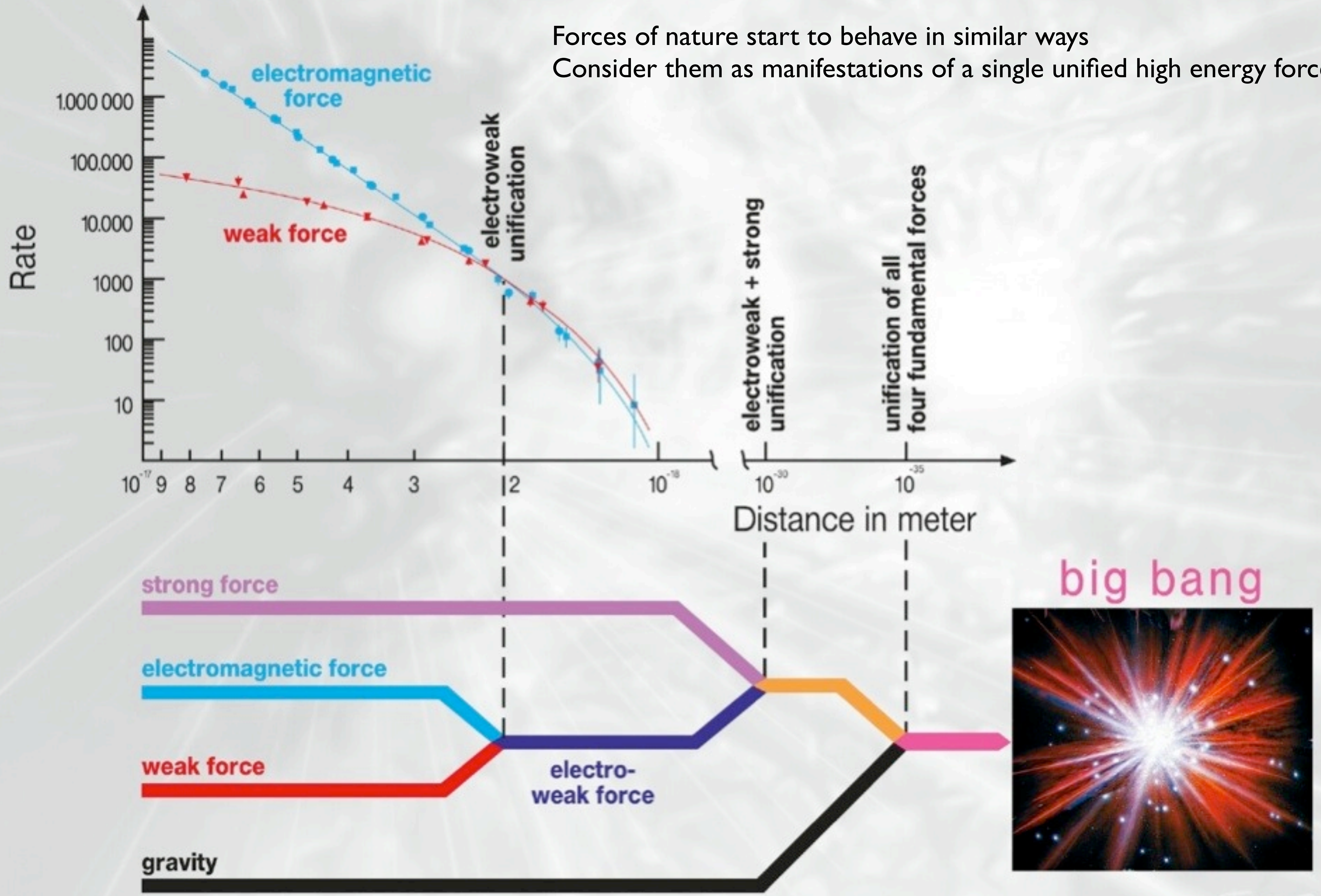
To measure the structure size x use wavelengths of similar size
- the probing scale

Don't use a finger to probe the structure of a sand grain!

Shorter wavelengths = higher energy
⇒ need more energetic colliders!

Higher energy → probing particle interactions further back in time
millionths of a second after the big bang

Forces of nature start to behave in similar ways
Consider them as manifestations of a single unified high energy force



Appendix

Wave functions and Operators

Macroscopic objects also have associated wave functions etc
But wavelength is immeasurably small!

$$\lambda = \frac{h}{p}$$

How is information about the particle 'encoded' in the wave function?
The wave function describes and contains all properties of the particle - denoted ψ

All measurable quantities are represented by a mathematical "operator" acting on the wave function

A travelling wave moving in space and time with definite momentum (fixed wavelength/frequency) can be written as:

$$\psi = A \sin\left(\frac{2\pi x}{\lambda} - \omega t\right)$$

A = amplitude of the wave
ω = frequency
λ = wavelength

We choose a position in space , x, and a time t and calculate the value of the wave function

Can also write this in the form: $\psi = Ae^{i(kx-\omega t)}$ and $k = \frac{2\pi}{\lambda}$ (ignore i for now)

If this represents the wave function of particle of definite (fixed) energy E then a measurement of energy should give us the answer E

We now posit that all measurements are represented by an operator acting on the wave function
Which mathematical operation will yield the answer E for the particle energy?

$$i\hbar\frac{\partial}{\partial t}$$

this is the derivative with respect to time
a derivative calculates the slope of a mathematical function
this is incomplete - it needs something to act on
just like + is incomplete without x and y to act on i.e. x+y
it acts on the wave function ψ

For a particle with wave function and definite energy E then:

$$i\hbar\frac{\partial}{\partial t}\psi = E\psi$$

This notation makes derivatives easier to calculate

$$\psi = Ae^{i(kx-\omega t)}$$

Similarly measurement of momentum for a particle with definite momentum p_x has the operator equation:

$$\frac{\hbar}{i}\frac{\partial}{\partial x}\psi = p_x\psi$$

In both cases the operator leaves the wave function unchanged
It is just multiplied by the momentum, or energy

(mathematically E and p are the eigenvalues of the equation)

The Schrödinger Equation

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = E\psi$$

operator for total kinetic energy
(energy by virtue of motion)
m = particle mass

For a free particle moving in 1 dimension with no forces acting on it and with definite energy:

1d Schrödinger equation
co-ordinate position x

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = i\hbar\frac{\partial}{\partial t}\psi \quad \xrightarrow{\text{compare this to classical equation}} \quad \frac{p^2}{2m} = E$$

Notice:
derivatives with respect to spatial co-ordinates are related to momenta
derivatives with respect to time co-ordinate is related to energy

In three dimensions (co-ordinate positions x,y,z):

$$\frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi = i\hbar\frac{\partial}{\partial t}\psi \longrightarrow \frac{-\hbar^2}{2m}\nabla^2\psi = i\hbar\frac{\partial}{\partial t}\psi$$

shorthand

Finally we include an interaction of the particle with an external (potential) energy field V

$$\frac{-\hbar^2}{2m}\nabla^2\psi + V(x,y,z)\psi = i\hbar\frac{\partial}{\partial t}\psi$$

this equation can now predict how particle moves / scatters under influence of the field V