Proton Structure at HERA

Measurements of Proton Structure at Low $Q^2$

The High $Q^2$ regime Neutral and Charged Current Processes

QCD: Partons in the Proton and $\alpha_s$

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Deep Inelastic Scattering

HERA collides e and p
study strong, electromagnetic & weak forces through Deep Inelastic Scattering

At fixed $\sqrt{s}$: two kinematic variables: $x$ & $Q^2$

$Q^2 = s \times y$

$Q^2$ = “resolving power” of probe
High $Q^2$: resolve $1/1000^{\text{th}}$ size of proton

$x =$ momentum fraction of proton carried by quark
HERA: $\sim 10^{-6} - 1$

Neutral Current Reaction

<table>
<thead>
<tr>
<th>Charged Current Reaction</th>
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<tbody>
<tr>
<td>e$^\pm$</td>
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<tr>
<td>$\nu_e$</td>
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<td>W$^\pm$</td>
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Eram Rizvi
Lake Louise Winter Institute – February 2004
Neutral and Charged Current Processes

\[
\frac{d\sigma_{NC}^\pm}{dx dQ^2} \approx \frac{e^4}{8\pi x} \left[ \frac{1}{Q^2} \right]^2 \left[ Y_+ \tilde{F}_2 + Y_- x\tilde{F}_3 - y^2 \tilde{F}_L \right]
\]

\[
\frac{d\sigma_{CC}^\pm}{dx dQ^2} \approx \frac{g^4}{64\pi x} \left[ \frac{1}{M_w^2 + Q^2} \right]^2 \left[ Y_+ \tilde{W}_2^\pm + Y_- x\tilde{W}_3^\pm - y^2 \tilde{W}_L^\pm \right]
\]

\[
Y_\pm = 1 \pm (1 - y)^2
\]

\[
\tilde{F}_2 \propto \sum (xq_i + x\bar{q}_i)
\]

\[
x\tilde{F}_3 \propto \sum (xq_i - x\bar{q}_i)
\]

\[
\tilde{F}_L \propto \alpha_s \cdot xg(x, Q^2)
\]

dominant contribution

only sensitive at high \(Q^2\)

only sensitive at low \(Q^2\)

and high \(y\)

similarly for \(W_2^\pm, xW_3^\pm\) and \(W_L^\pm\)

\[
\bar{\sigma}_{NC} = \frac{Q^2 x}{2\alpha_x^2} \frac{1}{Y_+} \frac{d^2\sigma}{dx dQ^2}
\]

\[
\bar{\sigma} = \tilde{F}_2 \text{ when } \tilde{F}_L \equiv x\tilde{F}_3 \equiv 0
\]
Conventional QCD evolution only tells us $Q^2$ dependence

$x$ dependence must come from data

**Method:**

- Measure cross sections
- Fit data – extract $x$ dep. of partons

HERA PDFs extrapolate into LHC region

LHC probes proton structure where gluon dominates (gluon collider)

HERA data crucial in calculations of new physics & measurements at LHC
F₂ dominates cross-section

Range in x: 0.00001 – 1

Range in Q² ~1 - 30000 GeV²

Measured with ~2-3% precision

Directly sensitive to sum of all quarks and anti-quarks

Indirectly sensitive to gluons via QCD radiation - scaling violations
Dramatic scaling violations at low $x$ driven by gluon described by QCD.
At low $Q^2$ cannot measure $xg(x,Q^2)$ via scaling violations of $F_2$.

$F_L$ is directly sensitive to gluon.

$\sigma_{NC}$ is sensitive to $F_L$ only at high $y$.

$\sigma_{FIT} = F_2 - \frac{y^2}{Y_+} F_L$

$F_2 = c \cdot x^{-\lambda}$

Shape of $\sigma_r$ at high $y$ driven by kinematic factor $y^2/Y_+$, not $F_L$ behaviour.

Whole $x$-range of measured data used to fit $F_2$ and $F_L$.

No extrapolation of $F_2$ - smaller errors.
Initial state radiation reduces $\sqrt{s}$

At fixed $x, Q^2$ then $y$ is different

Changes contribution of $F_2$ and $F_L$

Measure $\sigma_{NC}$ vs $y$: fit for $F_L$

\[ \sigma_{NC} = F_2 - \frac{y^2}{Y^+} F_L \]
xF3 and the valence quarks

At high Q2 NC cross sections for e⁺ and e⁻ deviate

\[ \tilde{\sigma}_NC^{\pm} \sim \tilde{F}_2 \mp \frac{Y}{Y_+} x\tilde{F}_3 \]

Subtract NC positron from electron cross section

HERA confirm valence quark structure

Errors dominated by stat. error of e- sample

Clear need for high luminosity
Charged current process provides sensitivity to quark flavour

Cross sections small due to large W mass in propagator

At high x (low y) lepton charge separates u from d

\[
\sigma^+_{cc} \approx x \left[ \bar{u} + \bar{c} + (1 - y)^2 (d + s) \right]
\]

\[
\sigma^-_{cc} \approx x \left[ u + c + (1 - y)^2 (\bar{d} + \bar{s}) \right]
\]
QCD analyses require many choices to be made
Should be reflected in PDF uncertainty:

- $Q_0^2$ starting scale
- Choice of data sets used
- Cuts to limit analysis to perturbative phase space ($Q^2_{\text{min}}$)
- Choice of densities to parameterise (e.g. $u_\nu$, $d_\nu$, $xg$, $xS$)
- Treatment of heavy quarks
- Allowed functional form of PDF parameterisation
- Treatment of experimental systematic uncertainties
- Renormalisation / factorisation scales
- Choice of $\alpha_s$
- etc...
PDFs parameterised at starting scale $Q_0^2$

$$xPDF(x, Q_0^2) = Ax^b (1-x)^c (1+dx+e\sqrt{x}+fx^2+gx^3)$$

Parameters $A, b, c, d, e, f$ optimised in fit for each PDF.

Choosing parameterisation is something of an art!

Unclear how to include choice of parameterisation in PDF uncertainty.

Some parameters constrained by sum rules e.g. momentum sum = 1:

$$\int u_v dx = 2 \quad \int d_v dx = 1$$
• ZEUS perform a new global analysis - use world structure function data
  
  **ZEUS 96/97 NC e^+ reduced cross sections**  
  Gluon / quarks at low x / Q^2

  **F₂ NMC p & D and ratio F₂ D/p**  
  Quarks at medium x

  **F₂ E665 p & D**  
  Quarks at medium x

  **F₂ BCDMS p only**  
  U quarks at high x / low Q^2

  **xF₃ CCFR (0.1 < x < 0.65)**  
  Valence quarks at high x / low Q^2

• Standard xg, xuᵥ, xdᵥ, Sea, x(\bar{d} – \bar{u}) decomposition of proton

• Q^2₀ = 7 GeV^2 / Q^2_{min} = 2.5 GeV^2

• Use functional form = A . x^b . (1-x)^c . (1 + dx + ex)

• Additional constraints on valence quark parameters (b_{uv} = b_{dv} = 0.5)

• Experimental systematic uncertainties are propagated onto final PDF uncertainty

• Use Thorne/Roberts variable flavour number scheme.

• x(\bar{d} – \bar{u}) params taken from MRST - only normalisation free in fit
Use only H1 inclusive NC & CC x-sections (e\(^+\)p and e\(^-\)p)

H1 dedicated fit: tune fitted PDFs to HERA NC/CC cross section sensitivity:

\[ xU = xu + xc \]
\[ xD = xd + xs \]
\[ xU = x\bar{u} + x\bar{c} \]
\[ xD = x\bar{d} + x\bar{s} \]
\[ xg \]

\[ u_v = U - \bar{U} \]
\[ d_v = D - \bar{D} \]

\[ F_2 = \frac{4}{9}(xU + xU) + \frac{1}{9}(xD + xD) \]

\[ \sigma_{cc}^+ = xU + (1 - y)^2 xD \]
\[ \sigma_{cc}^+ = xU + (1 - y)^2 xD \]

Perform fit in massless scheme - appropriate for high \(Q^2\)

Careful choice of parameterisations search for \(\chi^2\) saturation

\(Q^2_0 = 4\ \text{GeV}^2 / Q^2_{\text{min}} = 3.5\ \text{GeV}^2\)

Use BCDMS p and D data as cross check only

Similar technique used for dedicated H1+BCDMS fit for gluon + \(\alpha_s\)

Check for consistency of H1 & BCDMS datasets first
HERA PDFs

H1: $\chi^2 / \text{ndf} = 0.88$ (621 data points, 10 pars.)

ZEUS: $\chi^2 / \text{ndf} = 0.95$ (1263 data points, 11 pars.)

HERA able to extract PDFs w/o external input

PDFs broadly agree at low x (HERA data)

Discrepancies in med-high x region

CTEQ6 lies between the two fits

Some uncertainties unaccounted?
  - data sets inconsistent?
  - missing theory
  - PDF parametric forms?
  - different assumptions
H1: 0.1150 ±0.0017 (exp)  
+0.0009 -0.0007 (model)

if: systematic errors are not fitted: +0.0005

NMC replaces BCDMS 0.116±0.003 (exp)

4 light flavours: +0.0003

BCDMS deuteron data added: 0.1158 ± 0.0016 (exp)

ZEUS: 0.1166 ±0.0008 (unc) ±0.0032 (corr) ±0.0036 (norm) ±0.0018 (model)

systematic errors are not allowed to vary in chi2 minimisation

$Q^2 > 2.5 \text{ GeV}^2, \quad W^2 > 20 \text{ GeV}^2, \quad \text{RT-VFNS}, \quad b_{uv} = b_{dv} = 0.5$

fit $\alpha_s, xg, uv, dv,$ sea, dbar-ubar (MRST)

if fixed flavour scheme is used: +0.0010
Large $\chi^2$ variations if renormalisation scale is varied: $\pm 20$ units!

- $\to (\frac{1}{4} .. 4) : \pm 0.005$ (H1)
- $\to (\frac{1}{2} .. 2) : \pm 0.004$ (ZEUS)

Variations of factors 2 or 4 are ad hoc

Largest single uncertainty in determination of $\alpha_s$

Expected to be reduced in a NNLO analysis
NC: $Q^2 = 13500 \text{ GeV}^2$

HERA-II now in production Lumi mode
Experiments no longer limited by background
Machine routinely delivering almost 2 pb$^{-1}$ per week
Analysis of new data in progress...

High $Q^2$ events recorded recently in new data

CC: $Q^2 = 6000 \text{ GeV}^2$