Part C
The results and their interpretation

Chapter 16
The CKM matrix and the Kobayashi-Maskawa mechanism

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16.1 Historical background

Fundamentals

In the early twentieth century the "elementary" particles known were the proton, the electron and the photon. The first extension of this set of particles occurred with the neutrino hypothesis, first formulated by W. Pauli in his famous letter to his "radioactive friends" in 1924. From the theoretical side, the formulation of a theory of weak interactions by Fermi in 1934 marked another milestone in the development of our understanding. This set up for the first time a framework, in which some of the fundamental questions on the role of hadrons versus leptons and on the properties of particles and their interactions could be formulated. This also resulted in a clear formulation of "weak" versus "strong" interactions and the understanding of interactions as an exchange of mediating particles. In particular, Yukawa postulated the existence of such a particle and triggered the search for what we now know as the pion. At about the same time the muon was discovered, and initially called the "μ meson", however this soon turned out to be distinct from the pion.

Although the the term "flavor" came much later, one may mark the beginning of (quark) flavor physics by the discovery of strange particles (Rochester and Butler, 1947). Their decays into non-strange particles had lifetimes too long to be classified as strong decays: this led to the introduction of the strangeness quantum number (Gell-Mann, 1953), which is conserved in strong decays but may change in a weak decay.

The subsequent proliferation of new particles could nicely be classified and ordered by Gell Mann's "eightfold way" (Gell-Mann, 1962), which was an extension of the isospin symmetry to a symmetry based on the group $SU(3)$. However, none of the particles fitted into the fundamental representation of this group, although there were various attempts such as Sakata's model, in which the proton, the neutron and the Λ baryon formed the fundamental representation. Eventually this puzzle was resolved by the postulate of quarks as the fundamental building blocks of matter.

Strangeness, parity violation, and charm

The decays of the strange particles, in particular of the kaons, paved the way for the further development of our understanding. Before 1954, the three discrete symmetries C (charge conjugation), P (parity) and T (time reversal) were believed to be conserved individually, a conclusion drawn from the well known electromagnetic interaction. Based on this assumption, the so called θ-τ puzzle emerged: Two particles (at that time called θ and τ, where the latter is not to be confused with the third generation lepton) were observed, which had identical masses and lifetimes. However, they obviously had different parities, since the θ particle decayed into two pions (a state with even parity), and the τ particle decays into three pions (a state with odd parity).

The resolution was provided by the bold assumption by Lee and Yang (1956) that parity is not conserved in weak interactions, and θ and τ are in fact the same particle, which we now call the charged kaon. Subsequently the parity violating $V-A$ structure of the weak interaction was established and, on the experimental side, parity violation was confirmed directly in β decays (Garwin, Lederman, and Weinrich, 1957; Wu, Ambler, Hayward, Hoppes, and Hudson, 1957). However, the combination of two discrete transformations, namely CP, still seemed to be conserved.

Another puzzle related to kaon decays was the relative coupling strength. It tuned out that the coupling strength of strangeness-changing processes is much smaller than that of strangeness-conserving transitions. This finding eventually led to the parameterization of quark mixing by Cabibbo (1963). In modern language, the up quark u couples to a combination $d\cos\theta_C + s\sin\theta_C$ of the down quark d and the strange quark s. The value $\theta_C \sim 13^\circ$ for the Cabibbo angle explained the observed pattern of branching ratios in baryon decays.

Experiments at that time only probed the three lightest quarks, and there was no known reason for the extreme suppression of the flavor changing neutral current (FCNC) decay $K^+ \to \pi^+\ell^+\ell^-$ with respect to the charged current decay $K^+ \to \pi^0\ell^+\overline{\nu}$, $\Gamma(K^+ \to \pi^+\ell^+\ell^-)/\Gamma(K^+ \to \pi^0\ell^+\overline{\nu}) \sim 10^{-6}$. The resolution of this puzzle was found by Glashow, Iliopoulos, and Maiani (1970): one includes the charm quark, with the same quantum numbers as the up quark, and coupling to the orthogonal combination $-d\sin\theta_C + s\cos\theta_C$.

FCNC processes are suppressed by this "GIM mechanism". In fact, FCNC's in the kaon system involve a transition of an s quark into a d quark. This can be achieved by two successive charged current processes involving (in the two-family picture) either the up or the charm quark as an intermediate state. Taking Cabibbo mixing into account, these amplitudes are

$$\begin{aligned}\mathcal{A}(s \to d) &= \mathcal{A}(s \to u \to d) + \mathcal{A}(s \to c \to d) \\ &= \sin\theta_C \, \cos\theta_C \, [f(m_u) - f(m_c)],\end{aligned} \quad (16.1.1)$$

where $f(m)$ is some smooth function of the mass m. Hence, if the up and charm quark masses were degenerate, $K^0 - \overline{K}^0$ mixing and other kaon FCNC processes would not occur.

However, the up and charm masses are not degenerate and thus $K^0 - \overline{K}^0$ mixing can occur. Neglecting the small up-quark mass, the mixing amplitude turns out to be

$$\mathcal{A}(K \to \overline{K}) \propto \sin^2\theta_C \, \cos^2\theta_C \, \frac{m_c^2}{M_W^2}. \quad (16.1.2)$$

This implies that a mass difference Δm_K appears in the neutral kaon system. From this mass difference (an expression analogous to Eq. 10.1.17) Gaillard and Lee (1974b) could extract the prediction that the charm-quark mass should be about $m_c \sim 1.5$ GeV, and it was one of the great triumphs of particle physics when narrow resonances with masses of about 3 GeV were discovered a few months later (Aubert et al., 1974; Augustin et al., 1974): these were identified as $c\overline{c}$ bound states. Around this time the term "particle family" was coined, and the discovery of the charm quark completed the second particle family; it also introduced a 2×2 quark mixing matrix into the phenomenology of weak interactions.

CP violation and the Kobayashi-Maskawa mechanism

Almost ten years before the discovery of charm, *CP* violation was observed in the study of rare kaon decays by Christenson, Cronin, Fitch, and Turlay (1964). This effect is difficult to accommodate for two families, but an extension to three families allows it to be taken into account naturally. The "six-quark model" was proposed by Kobayashi and Maskawa (1973), extending Cabibbo's 2×2 quark mixing matrix into the 3×3 Cabibbo-Kobayashi-Maskawa (CKM) matrix. The GIM mechanism for the six quark model is implemented by the unitarity of the CKM matrix.

While the observation of decays $K_L^0 \to 2\pi$ meant that *CP* was violated, the data at that time only required *CP* violation in mixing (see Section 16.6 for the classification of *CP*-violating effects). The observed strength of *CP* violation in mixing, $\varepsilon_K \simeq 2.3 \times 10^{-3}$, was consistent with the Kobayashi-Maskawa (KM) mechanism (Ellis, Gaillard, and Nanopoulos, 1976; Pakvasa and Sugawara, 1976). However, this did not constitute a proof that the KM mechanism was really the origin of the observed *CP* violation; the measurement of the single parameter ε_K could not be used to test the KM mechanism. One alternative explanation was offered by the super-weak model of Wolfenstein (1964), where *CP* violation was due to a new, very weak four-fermion interaction that changed strangeness by 2 units ($\Delta S = 2$). This possibility was ruled out by the observation of direct *CP* violation in $K_L \to \pi\pi$ decays, $\mathrm{Re}(\varepsilon'_K/\varepsilon_K) = (1.65 \pm 0.26) \times 10^{-3}$ (Alavi-Harati et al., 1999; Burkhardt et al., 1988; Fanti et al., 1999). Nonetheless, convincing evidence for the KM mechanism required the measurement of $\sin(2\phi_1)$ at the B Factories.

With the discovery of the τ lepton in 1975 (Perl et al., 1975) and of the bottom quark in 1977 (Herb et al., 1977) it became clear that there is a third generation of quarks and leptons. Furthermore, the bottom quark turned out to be quite long-lived, indicating a small mixing angle between the first and second generation. This fact is the experimental foundation of using B decays to study *CP* violation, as well as for b tagging in high-p_t physics.

The third generation remained incomplete for many decades, since the top quark turned out to be quite heavy, and a direct discovery had to wait until 1995, when it was discovered at the Tevatron at Fermilab (Abachi et al., 1995a; Abe et al., 1994). However, the first hint of the large top-quark mass was the discovery of $B^0 - \overline{B}^0$ oscillations (also known as mixing) by ARGUS (Albrecht et al., 1987b). The measured Δm_d implied a heavy top with a mass m_t above $50 - 70$ GeV, if the standard six quark model was assumed (Bigi and Sanda, 1987; Ellis, Hagelin, and Rudaz, 1987). The phenomenon of neutral meson mixing is discussed in Chapter 10, while Section 17.5 discusses results on B mixing from the B Factories.

In fact, if the top mass had been significantly smaller, ARGUS could not have observed $B^0 - \overline{B}^0$ oscillations. The GIM mechanism for down-type quarks leads generally to suppression factors of the form

$$\text{CKM Factor} \times \frac{1}{16\pi^2} \frac{m_t^2 - m_u^2}{M_W^2} \quad (16.1.3)$$

and hence the GIM suppression for the bottom quark is much weaker than in the up-quark sector, where the corresponding factor is

$$\text{CKM Factor} \times \frac{1}{16\pi^2} \frac{m_b^2 - m_d^2}{M_W^2}. \quad (16.1.4)$$

Hence FCNC decays of B-mesons have branching ratios in the measurable region, while FCNC processes for D-mesons are heavily suppressed.

The third particle family was completed by the discovery of the τ neutrino as a particle distinct from the electron and the muon neutrino by the DONUT collaboration (Kodama et al., 2001). Although models with a fourth particle generation are frequently considered as benchmark models for physics beyond the Standard Model, there is no indication of a fourth family. On the contrary, from the width of the Z boson precisely measured at LEP it can be inferred that there is no further family with a neutrino lighter than 40 GeV, and the recent discovery of a Higgs boson in the mass range of 125 GeV (Aad et al., 2012; Chatrchyan et al., 2012b) rules out a large class of fourth-generation models.

16.2 *CP* violation and baryogenesis

Particle physics experiments of the past thirty years have confirmed the Standard Model (SM) even at the quantum level, including quark mixing and *CP* violation. However, the observed matter-antimatter asymmetry of the universe indicates that there must be additional sources of *CP* violation, since the amount of *CP* violation implied by the CKM mechanism is insufficient to create the observed matter-antimatter asymmetry.

In fact, the excess of baryons over antibaryons in the universe

$$\Delta = n_{\mathfrak{B}} - n_{\overline{\mathfrak{B}}} \tag{16.2.1}$$

is small compared to the number of photons: the ratio is measured to be $\Delta/n_\gamma \sim 10^{-10}$. Although it is conceivable that there might be regions in the universe consisting of antimatter, just as our neighborhood consists of matter, no mechanism is known which could, from the Big Bang, produce regions of matter (or antimatter) as large as we observe today. Furthermore, searches have been performed for sources of photons indicative of regions of matter and antimatter colliding. These searches failed to find any large regions of antimatter.

The conditions under which a non-vanishing Δ can emerge dynamically from the symmetric situation $\Delta = 0$ have been discussed by Sakharov (1967). He identified three ingredients

1. There must be baryon number violating interactions $H_{\text{eff}}(\Delta\mathfrak{B} \neq 0) \neq 0$.
2. There must be *CP* violating interactions. If *CP* were unbroken, then we would have for every process $i \to f$ mediated by $H_{\text{eff}}(\Delta\mathfrak{B} \neq 0)$ the *CP* conjugate one with the same probability
$$\Gamma(i \to f) = \Gamma(\bar{i} \to \overline{f}) \tag{16.2.2}$$
which would erase any matter-antimatter asymmetry.
3. The universe must have been out of thermal equilibrium. Under the assumption of locality, causality, and Lorentz invariance, *CPT* is conserved. Since in an equilibrium state time becomes irrelevant on the global scale, *CPT* reduces to *CP*, and the argument of point 2 applies.

In order to illustrate the first two Saharov conditions, we employ a very simplistic example. Assume that in the early universe, there was a particle X that could decay to only two final states $|f_1\rangle$ and $|f_2\rangle$, with baryon numbers $N_{\mathfrak{B}}^{(1)}$ and $N_{\mathfrak{B}}^{(2)}$ respectively, and decay rates

$$\Gamma(X \to f_1) = \Gamma_0 r \quad \text{and} \quad \Gamma(X \to f_2) = \Gamma_0(1-r)\,, \tag{16.2.3}$$

where Γ_0 is the total width of X. Taking the *CP* conjugate, the particle $\overline{X}$ decays to the state $\overline{f}_1$ with baryon number $-N_{\mathfrak{B}}^{(1)}$ and $\overline{f}_2$ with baryon number $-N_{\mathfrak{B}}^{(2)}$; the rates are

$$\Gamma(\overline{X} \to \overline{f}_1) = \Gamma_0 \overline{r} \quad \text{and} \quad \Gamma(\overline{X} \to \overline{f}_2) = \Gamma_0(1-\overline{r}), \tag{16.2.4}$$

where Γ_0 is the same as for X due to *CPT* invariance.

The overall change $\Delta N_{\mathfrak{B}}$ in baryon number induced by the decay of an equal number of X and $\overline{X}$ particles is

$$\begin{aligned} \Delta N_{\mathfrak{B}} &= r N_{\mathfrak{B}}^{(1)} + (1-r) N_{\mathfrak{B}}^{(2)} - \overline{r} N_{\mathfrak{B}}^{(1)} - (1-\overline{r}) N_{\mathfrak{B}}^{(2)} \\ &= (r - \overline{r}) \left(N_{\mathfrak{B}}^{(1)} - N_{\mathfrak{B}}^{(2)} \right) \end{aligned} \tag{16.2.5}$$

Thus $\Delta N_{\mathfrak{B}} \neq 0$ means that we have to have *CP* violation ($r \neq \overline{r}$) and a violation of baryon number ($N_{\mathfrak{B}}^{(1)} \neq N_{\mathfrak{B}}^{(2)}$), illustrating the first two conditions.

Sakharov's paper remained mostly unnoticed until the first formulation of Grand Unified Theories (GUTs). In these theories, for the first time, all the necessary ingredients were present. In particular, baryon number violation appears naturally since quarks and leptons appear in the same multiplets of the GUT symmetry group. Furthermore, there are additional sources of *CP* violation, and a phase transition takes place at the scale M_{GUT}, which has to be quite high to prevent proton decay.

One may also consider electroweak baryogenesis. The electroweak interaction provides *CP* violation through the CKM mechanism, and the electroweak phase transition has been thoroughly studied. The first ingredient is also present, as the current corresponding to baryon number is conserved only at the classical level: electroweak quantum effects violate baryon number, but still conserve the difference $\mathfrak{B} - L$ of baryon and lepton number. However, although all the ingredients are present, this cannot explain Δ. In particular, the CKM *CP* violation is too small by several orders of magnitude.

Given the firm evidence for non-vanishing neutrino masses, there could be new sources of *CP* violation in the lepton sector, and even (although there is no evidence for this as yet) lepton-number violation. This could lead to violation of baryon number via leptogenesis, with the surplus of leptons transferred to the baryonic sector through $(\mathfrak{B} - L)$-conserving interactions.

In any case, an additional source(s) of *CP* violation is needed, beyond the phase of the CKM matrix (which is explained in the next section), in order to explain the matter-antimatter asymmetry of the universe. The search for this new interaction is one of the main motivations for flavor-physics experiments.

16.3 *CP* violation in a Lagrangian field theory

The SM is formulated as a quantum field theory based on a Lagrangian derived from symmetry principles. To this end, the (hermitian) Lagrangian of the SM is given in terms of scalar operators $\mathcal{O}_i$ with couplings a_i

$$\mathcal{L}(x) = \sum_i \left(a_i \mathcal{O}_i(x) + a_i^* \mathcal{O}_i^\dagger(x) \right)\,, \tag{16.3.1}$$

where the $\mathcal{O}_i$ are composed of the SM quark, lepton, and gauge fields. It is straightforward to verify that *CP* conservation implies that all couplings a_i can be made real by suitable phase redefinitions of the fields composing the

$\mathcal{O}_i$. In turn, *CP* is violated in a Lagrangian field theory if there is no choice of phases that renders all a_i real.

In the SM there are in principle two sources of *CP* violation. The so-called "strong *CP* violation" originates from special features of the QCD vacuum, resulting in a contribution of the form

$$\mathcal{L}_{\text{strong}\,CP} = \theta \, \frac{\alpha_S}{8\pi} G^{\mu\nu,a} \tilde{G}^a_{\mu\nu} \tag{16.3.2}$$

where $G_{\mu\nu}$ ($\tilde{G}_{\mu\nu}$) is the (dual) strength of the gluon field. This term is *P* and *CP* violating due to its pseudoscalar nature. However, a term such as Eq. (16.3.2) will have a strong impact on the electric dipole moment (EDM) of the neutron, $d_N \sim \theta \times 10^{-15}$ e cm. In combination with the current limit on the neutron EDM of $d_N < 0.29 \times 10^{-25}$ e cm, this yields a stringent limit, $\theta \leq 10^{-10}$. However, the theoretical reason for its smallness has not yet been discovered. This is known as the "strong *CP* problem" (see for example Cheng, 1988; Kim and Carosi, 2010); we shall ignore this in what follows by setting $\theta = 0$.

The second source of *CP* violation is the CKM matrix. It turns out that all terms in the SM Lagrangian are *CP* invariant except for the charged current interaction term

$$H_{\text{cc}} = \frac{g}{\sqrt{2}} \left(\overline{u}_L \, \overline{c}_L \, \overline{t}_L \right) V_{\text{CKM}} \gamma^\mu \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W^+_\mu . \tag{16.3.3}$$

Under a *CP* transformation we have

$$\left(\overline{u}_L \, \overline{c}_L \, \overline{t}_L \right) V_{\text{CKM}} \gamma^\mu \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W^+_\mu \tag{16.3.4}$$

$$\xrightarrow{CP} \left(\overline{d}_L \, \overline{s}_L \, \overline{b}_L \right) V^T_{\text{CKM}} \gamma^\mu \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} W^-_\mu \tag{16.3.5}$$

and hence the combination $H_{\text{cc}} + H^\dagger_{\text{cc}}$ appearing in the SM Lagrangian is *CP* invariant, if

$$V^T_{\text{CKM}} = V^\dagger_{\text{CKM}} \quad \text{or} \quad V_{\text{CKM}} = V^*_{\text{CKM}} . \tag{16.3.6}$$

This statement refers to a specific phase convention for the quark fields; in general terms it implies that in the *CP*-invariant case, the CKM matrix can be made real by an appropriate phase redefinition of the quark fields.

16.4 The CKM matrix

The CKM matrix V_{CKM} appearing in Eq. (16.3.3) is explicitly written as

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} . \tag{16.4.1}$$

Here the V_{ij} are the couplings of quark mixing transitions from an up-type quark $i = u, c, t$ to a down-type quark $j = d, s, b$.

In the SM the CKM matrix is unitary by construction. Using the freedom of phase redefinitions for the quark fields, the CKM matrix has $(n-1)^2$ physical parameters for the case of n families. Out of these, $n(n-1)/2$ are (real) rotation angles, and $((n-3)n+2)/2$ are phases, which induce *CP* violation. For $n = 2$, no *CP* violation is possible, while for $n = 3$ a single phase appears. This is the unique source of *CP* violation in the SM, once the possibility of strong *CP* violation is ignored.

The CKM matrix for 3 families may be represented by three rotations and a matrix generating the phase

$$U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} ,$$

$$U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix} ,$$

$$U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} ,$$

$$U_\delta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix} , \tag{16.4.2}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, and δ is the complex phase responsible for *CP* violation; by convention the mixing angles θ_{ij} are chosen to lie in the first quadrant so that the s_{ij} and c_{ij} are positive. Then (Chau and Keung, 1984)

$$\begin{aligned} V_{\text{CKM}} &= U_{23} U^\dagger_\delta U_{13} U_\delta U_{12} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} . \end{aligned} \tag{16.4.3}$$

This is the representation used by the PDG (Beringer et al., 2012).

The elements of the CKM matrix exhibit a pronounced hierarchy. While the diagonal elements are close to unity, the off-diagonal elements are small, such that *e.g.* $V_{ud} \gg V_{us} \gg V_{ub}$. In terms of the angles θ_{ij} we have $\theta_{12} \gg \theta_{23} \gg \theta_{13}$. This fact is usually expressed in terms of the Wolfenstein parameterization (Wolfenstein, 1983), which can be understood as an expansion in $\lambda = |V_{us}|$. It reads up to order λ^3

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \tag{16.4.4}$$

The parameters A, ρ and η are assumed to be of order one. When using this parameterization, one has to keep in mind that unitarity is satisfied only up to order λ^4. As it turns out that both ρ and η are also of order λ, the extension to higher orders becomes non-trivial, and one has to consider redefining the parameters accordingly; this has been studied by Ahn, Cheng, and Oh (2011).

One can obtain an exact parameterization of the CKM matrix in terms of A, λ, ρ, and η, for example, by following the convention of Buras, Lautenbacher, and Ostermaier (1994), where

$$\lambda = s_{12}, \tag{16.4.5}$$
$$A = s_{23}/\lambda^2, \tag{16.4.6}$$
$$A\lambda^3(\rho - i\eta) = s_{13}e^{-i\delta}, \tag{16.4.7}$$

and by substituting Eqs (16.4.5) through (16.4.7) into Eq. (16.4.3), while noting that $\sin^2\theta = 1 - \cos^2\theta$. Such a parameterization is described in Section 19.2.1.3 to illustrate *CP* violation in the charm sector.

Sometimes a slightly different convention for the Wolfenstein parameters is used, with parameters denoted $\overline{\rho}$ and $\overline{\eta}$. These parameters were defined at fixed order by Buras, Lautenbacher, and Ostermaier (1994); the modern definition (Charles et al., 2005),

$$\overline{\rho} + i\overline{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}, \tag{16.4.8}$$

holds to all orders. The difference with the parameterization defined above appears only at higher orders in the Wolfenstein expansion; the relation between this scheme and the one defined in (16.4.5–16.4.7) is given by

$$\rho + i\eta = (\overline{\rho} + i\overline{\eta})\frac{\sqrt{1 - A^2\lambda^4}}{\sqrt{1-\lambda^2}[1 - A^2\lambda^4(\overline{\rho} + i\overline{\eta})]}. \tag{16.4.9}$$

16.5 The Unitarity Triangle

The unitarity relations $V_{\rm CKM} \cdot V_{\rm CKM}^\dagger = 1$ and $V_{\rm CKM}^\dagger \cdot V_{\rm CKM} = 1$ yield six independent relations corresponding to the off-diagonal zeros in the unit matrix. They can be represented as triangles in the complex plane; each triangle has the same area, reflecting the fact that (with three families) there is only one irreducible phase. A non-trivial triangle — one with angles other than 0 or π — indicates *CP* violation, proportional to the triangles' common area. Bigi and Sanda (2000) provide a detailed discussion of the various triangles, their interpretation, and the possibilities to probe them. Only two triangles have sides of comparable length, which means that they are of the same order in the Wolfenstein parameter λ. The corresponding relations are

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \tag{16.5.1}$$
$$V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0. \tag{16.5.2}$$

Inserting the Wolfenstein parameterization, both relations turn out to be identical, up to terms of order λ^5; the apex of the Unitarity Triangle is given by the coordinate $(\overline{\rho}, \overline{\eta})$. The three sides of this triangle (Fig. 16.5.1) — usually referred to as "the" Unitarity Triangle— control semi-leptonic and non-leptonic B_d transitions, including $B_d - \overline{B}_d$ oscillations. In order to obtain the triangle shown in Fig 16.5.1, Eq. (16.5.1) is divided by $V_{cd}V_{cb}^*$ so that the base of the triangle is of unit length. Due to the sizable angles, one expects large *CP* asymmetries in B decays in the SM; this was actually realized *before* the discovery of "long" B lifetimes. Note that in both unitarity-triangle relations CKM matrix elements related to the top quark appear; in particular V_{td} and V_{ts} can be accessed only indirectly via FCNC decays of bottom quarks.

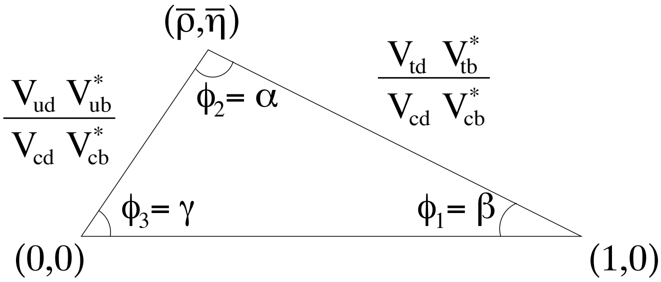

Figure 16.5.1. The Unitarity Triangle.

The angles of the Unitarity Triangle are defined as

$$\phi_1 = \beta \equiv \arg\left[-V_{cd}V_{cb}^*/V_{td}V_{tb}^*\right], \tag{16.5.3}$$
$$\phi_2 = \alpha \equiv \arg\left[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*\right], \tag{16.5.4}$$
$$\phi_3 = \gamma \equiv \arg\left[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*\right], \tag{16.5.5}$$

where this definition is independent of the specific phase choice expressed in Eq. (16.4.3). Different notation conventions have been used in the literature for these angles. In particular the *BABAR* experiment has used α, β, and γ, whereas the Belle experiment has reported results in terms of ϕ_2, ϕ_1, and ϕ_3, respectively. We use the latter for brevity when discussing results in later sections.

The presence of *CP* violation in the CKM matrix implies non-trivial values for these angles ($\phi_i \neq 0°$, $180°$), corresponding to a non-vanishing area for the Unitarity Triangle. In fact, all the triangles that can be formed from the unitarity relation have the same area, which is proportional to the quantity

$$\Delta = {\rm Im}\, V_{cs}^*V_{us}V_{cd}V_{ud}^* \tag{16.5.6}$$

which is independent of the phase convention. Note that all other, rephasing invariant fourth order combinations of CKM matrix elements, which cannot be reduced to products of second order invariants, can be related to Δ, which is thus unique.

Furthermore, the phase in the CKM matrix could also be removed, if the masses of either two up-type quarks or two down-type quarks were degenerate. In summary, the presence of *CP* violation is equivalent to (Jarlskog, 1985)

$$\begin{aligned} J &= \det[M_u\,,\, M_d] \\ &= 2i\Delta \times (m_u - m_c)(m_u - m_t)(m_c - m_t) \\ &\qquad \times (m_d - m_s)(m_d - m_b)(m_s - m_b) \end{aligned} \tag{16.5.7}$$

being non-vanishing.

The SM allows us to construct "the" Unitarity Triangle by measuring its angles or its sides or any combinations of them. Any discrepancy between the observed and predicted values indicates a manifestation of dynamics beyond the SM. Clearly this requires good control of experimental and theoretical uncertainties, both in their *CP* sensitive and insensitive rates.

Measurements of the magnitudes of CKM matrix elements V_{ub} and V_{cb} can be found in Section 17.1, and measurements of V_{td} and V_{ts} in Section 17.2. Measurements of the angles ϕ_1, ϕ_2, and ϕ_3 are discussed in Sections 17.6, 17.7, and 17.8 respectively. It is possible to perform *global fits*, using data from many decay processes to over-constrain our knowledge of the CKM mechanism. Given the lack of knowledge of the determination of the apex of the Unitarity Triangle, these global fits are often expressed in terms of constraints on the $(\overline{\rho}, \overline{\eta})$ plane. Some experimental results require input from Lattice QCD calculations in order to be used in a global fit. These global fits are discussed in Chapter 25, both in the context of the SM (Section 25.1) and allowing for physics beyond the SM (Section 25.2).

It is exactly some of the measurements described in Chapter 17 and further in Section 25.1 which were addressed in (Nobelprize.org, 2010) among experimental verifications of the Kobayashi-Maskawa mechanism in the scientific background to the 2008 Nobel Prize in Physics awarded to M. Kobayashi and T. Maskawa: "The respective collaborations BABAR and BELLE have now measured the *CP* violation in remarkable agreement with the model ... and all experimental data are now in impressive agreement with the model ...".

16.6 *CP* violation phenomenology for B mesons

Since *CP* violation is due to irreducible phases of coupling constants, it becomes observable through interference effects. The simplest example is an amplitude consisting of two distinct contributions

$$A_f = \lambda_1 \langle f|O_1|B\rangle + \lambda_2 \langle f|O_2|B\rangle \tag{16.6.1}$$

where $\lambda_{1,2}$ are (complex) coupling constants (in our case combinations of CKM matrix elements) and $\langle f|O_{1,2}|B\rangle$ are matrix elements of interaction operators between the initial and final state.

The *CP* conjugate is the process $\overline{B} \to \overline{f}$, yielding

$$\overline{A}_{\overline{f}} = \lambda_1^* \langle \overline{f}|O_1^\dagger|\overline{B}\rangle + \lambda_2^* \langle \overline{f}|O_2^\dagger|\overline{B}\rangle. \tag{16.6.2}$$

The matrix elements of $\mathcal{O}_{1,2}^{(\dagger)}$ involve only strong interactions, which we assume to be *CP*-invariant. Hence we have

$$\langle \overline{f}|O_1^\dagger|\overline{B}\rangle = \langle f|O_1|B\rangle \quad \text{and} \quad \langle \overline{f}|O_2^\dagger|\overline{B}\rangle = \langle f|O_2|B\rangle. \tag{16.6.3}$$

Thus for the *CP* asymmetry we find

$$\begin{aligned} \mathcal{A}_{CP}(B \to f) &\equiv \frac{\Gamma(B \to f) - \Gamma(\overline{B} \to \overline{f})}{\Gamma(B \to f) + \Gamma(\overline{B} \to \overline{f})} \\ &\propto 2\,\mathrm{Im}[\lambda_1 \lambda_2^*]\,\mathrm{Im}[\langle f|O_1|B\rangle \langle f|O_2|B\rangle^*]. \end{aligned} \tag{16.6.4}$$

Consequently, in order to create *CP* violation, there has to be — aside from the "weak phase" due to the complex phases of the CKM matrix — also a "strong phase", *i.e.* a phase difference between the matrix elements $\langle f|O_1|B\rangle$ and $\langle f|O_2|B\rangle$. In the SM these two contributions correspond to different diagram topologies. In many cases, one can identify tree-level contributions which carry different CKM factors compared to loop (penguin) contributions. *CP* violation then emerges from the interference of "trees" and "penguins".

In the following we are going to consider decays into CP eigenstates f in which case we have $f = \overline{f}$. For a quantum-coherent pair of neutral B-mesons (like the color-singlet $B^0\overline{B}^0$ pair from $\Upsilon(4S)$ decay) the time evolution generates a phase difference $\Delta m\,\Delta t$, which acts like the strong phase difference between the amplitudes for $B \to f$ and for $B \to \overline{B} \to f$. Hence we make use of the time-dependent *CP* asymmetry

$$\begin{aligned} \mathcal{A}_{\rm CP}^{B\to f}(\Delta t) &\equiv \frac{\Gamma(B^0(\Delta t) \to f) - \Gamma(\overline{B}^0(\Delta t) \to f)}{\Gamma(B^0(\Delta t) \to f) + \Gamma(\overline{B}^0(\Delta t) \to f)} \\ &= S^{B\to f}\, \sin(\Delta m_d\, \Delta t) - C^{B\to f}\, \cos(\Delta m_d\, \Delta t)\,. \end{aligned} \tag{16.6.5}$$

The derivation (see the discussion in Chapter 10 leading to Eq. 10.2.8) neglects the small lifetime difference $\Delta\Gamma$ in the B_d system; the expressions for S and C can be found in Eqs (10.2.4) and (10.2.5).

We may distinguish three different types of *CP* violation according to the various sources from which it emerges. *CP violation in decays*, sometimes referred to as *direct CP violation*, stems from different rates for a process and for its *CP* conjugate: hence we have $|\overline{A}_f/A_f| \neq 1$. This contribution leads to $C^{B\to f} \neq 0$: it is already present at $\Delta t = 0$, and remains in time-integrated measurements. *CP violation in the mixing* emerges in cases where we have $|p/q| \neq 1$.[49] One observable related to this is the semileptonic decay asymmetry $a_{\rm SL}$, which is the asymmetry between the decay rate of $B^0 \to X^- \ell^+ \nu_\ell$ and the *CP* conjugate process. Finally, *mixing-induced CP violation*, sometimes also called *CP violation in interference between a decay without mixing and a decay with mixing* occurs for $\mathrm{Im}\lambda \neq 0$, in which case interference of the amplitudes $B \to f$ and $B \to \overline{B} \to f$ leads to *CP* violation.[50]

[49] For a definition of the quantities p, q, and λ, we refer to Chapter 10, where time evolution is considered.

[50] In kaon physics sometimes the notion *indirect CP violation* is used for saying that the parameter ϵ is non-vanishing. Comparing this with the definitions given here, non-vanishing ϵ corresponds to a combination of $|q/p| \neq 1$ and $|\overline{A}_f/A_f| \neq 1$.

In the B_d system we have to a very good approximation[51]

$$\frac{q}{p} = \exp(-2i\phi_1)\ . \tag{16.6.6}$$

This follows from Eq. (10.1.19) and by inspection of the box diagram contributing to the B_d mixing (Fig. 10.1.1), from which it can be seen that the CKM matrix elements appearing in the amplitude yield $\phi_{M_{12}} = 2\phi_1$. Hence in all cases where $A = \overline{A}$, we find $|\lambda| = 1$ and $\mathrm{Im}\lambda = -\sin(2\phi_1)$, leading to

$$C^{B\to f} = 0 \quad \text{and} \quad S^{B\to f} = -\sin(2\phi_1). \tag{16.6.7}$$

This holds for the golden mode $B \to J/\psi K_s$ where there is no relative weak phase between A and $\overline{A}$. However, if there appears a relative weak phase in the decay amplitudes, then we may still have $|A| = |\overline{A}|$ and hence $|\lambda| = 1$, and thus no direct *CP* violation. For example, the tree amplitude in $B \to \pi\pi$ carries a weak phase $e^{-i\phi_3}$ which (neglecting penguin contributions) would lead to

$$\lambda = \exp(-2i(\phi_1 + \phi_3)) = \exp(+2i\phi_2). \tag{16.6.8}$$

However, the penguin contribution in $B \to \pi\pi$ cannot be neglected; in particular it leads to $|\lambda| \neq 1$ and to direct *CP* violation in these decays.

In general we have the "unitarity relation" between the quantities $S^{B\to f}$ and $C^{B\to f}$,

$$\left(C^{B\to f}\right)^2 + \left(S^{B\to f}\right)^2 = 1 - \left(D^{B\to f}\right)^2 \leq 1 \tag{16.6.9}$$

where

$$D^{B\to f} = \frac{2\,\mathrm{Re}\lambda}{1 + |\lambda|^2}. \tag{16.6.10}$$

However, in the limit of vanishing lifetime difference the time-dependent *CP* asymmetry does not depend on $D^{B\to f}$, and hence a direct measurement of this quantity in the B_d system is difficult.

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[51] This relation depends on the phase conventions used. It holds in the convention used in (16.4.3).