Flavour Physics and CP Violation

Post-FPCP 2018 Summer School
IIT Hyderabad, India

Lecture 3

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[Graph and Image]

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Outline

- Short recap
- Angles of the unitarity triangle
The Unitarity Triangle [recap]

\[ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \]

\[ \alpha = \pi - \beta - \gamma \]

\[ \rho + i\eta \]

\[ \gamma = \text{atan} \left( \frac{\eta}{\rho} \right) \]

\[ \beta = \text{atan} \left( \frac{\eta}{1 - \rho} \right) \]

many observables functions of \( \bar{\rho} \) and \( \bar{\eta} \): overconstraining
CPV Types for the B Meson System [recap]

Define the quantity $\lambda$:

$$\lambda_{f_{CP}} = \frac{q}{p} \cdot \frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}}$$

1. Indirect CP violation, or CPV in the mixing:
   $$| q/p | \neq 1$$

2. Direct CP violation, or CPV in the decays:
   $$| \bar{A}/A | \neq 1$$

3. CP violation in interference between mixing and decay:
   $$\text{Im}\lambda \neq 0$$

- both neutral and charged B
- neutral B
If we consider that both $B^0$ and $\bar{B}^0$ can decay to the same final state and considering here a final state that is a CP eigenstate, then the time evolution of the physical system becomes:

$$f(B^0_{phys} \to f_{CP}, \Delta t) = \frac{T}{4} e^{-T|\Delta t|} [1 - S_{f_{CP}} \sin (\Delta m_d \Delta t) + C_{f_{CP}} \cos (\Delta m_d \Delta t)]$$

$$f(\bar{B}^0_{phys} \to f_{CP}, \Delta t) = \frac{T}{4} e^{-T|\Delta t|} [1 + S_{f_{CP}} \sin (\Delta m_d \Delta t) - C_{f_{CP}} \cos (\Delta m_d \Delta t)]$$

- **direct CP violation**  \( C \neq 0 \)  \( C_f(= -A_f) = \frac{1 - |\lambda_{f_{CP}}|^2}{1 + |\lambda_{f_{CP}}|^2} \)

- **CP violation in interference**  \( S \neq 0 \)  \( S_f = \frac{2 \text{Im} \lambda_{f_{CP}}}{1 + |\lambda_{f_{CP}}|^2} \)
Theoretically cleaner (SM uncertainties $\sim 10^{-2}$ to $10^{-3}$) → tree dominated decays to Charmonium + $K^0$ final states.

\[ \beta \equiv \arg \left[ -V_{cd} V_{cb}^* / V_{td} V_{tb}^* \right] \]
\[ \beta \equiv \arg[-V_{cd}V^*_{cb}/V_{td}V^*_{tb}] \]

\[
\sin 2\beta \text{ in golden } b \to \bar{c}\bar{c}s \text{ modes }
\]

leading-order tree decays to \( \bar{c}\bar{c}s \) final states:

\[
\begin{array}{c}
\bar{b} \\
\downarrow V_{cb}
\end{array} \quad \begin{array}{c}
\bar{c} \\
\downarrow V_{*cs}
\end{array} \quad \begin{array}{c}
\bar{s} \\
\downarrow V_{cs}
\end{array}
\]

here the CKM elements contributing are \( V_{cb}V^*_{cs} \) that in our Wolfenstein CKM parameterisation have no phase.

The CP conjugated case is also leading to (about) the same final state:

\[
\begin{array}{c}
\bar{b} \\
\downarrow V^*_{cb}
\end{array} \quad \begin{array}{c}
\bar{c} \\
\downarrow V_{cs}
\end{array} \quad \begin{array}{c}
\bar{s} \\
\downarrow V_{cs}
\end{array}
\]

\[ \bar{B}^0 \to J/\psi K_{S,L} \]

\[ B^0 \to J/\psi K_{S,L} \]
$\beta \equiv \arg \left[-V_{cd}V_{cb}^{*}/V_{td}V_{tb}^{*}\right]$

$\sin 2\beta$ in golden $b \rightarrow c\bar{c}s$ modes

leading-order tree decays to $c\bar{c}s$ final states

$\bar{b} \rightarrow \bar{c} \rightarrow c$ + d \quad K^0 \rightarrow K_{S,L}$

because both $B$ and $\bar{B}$ can decay in this common final state, this can interfere with the oscillation diagram:

$\lambda = \frac{qA(\bar{B} \rightarrow f)}{pA(B \rightarrow f)} = \frac{V_{tb}^{*}V_{tb}^{*}}{V_{td}V_{td}^{*}} \frac{A}{A} \sim e^{-i2\beta} \frac{\tilde{A}}{A}$
\[ \beta \equiv \arg \left[ -V_{cd}V_{cb}^{*}/V_{td}V_{tb}^{*} \right] \]

\[ \sin^2 \beta \text{ in golden } b \to ccs \text{ modes} \]

\[ B^0 \to J/\psi K_{S,L} \]

\[ \lambda_{CP} = \eta_{CP} \frac{q}{p} \frac{A}{A} = \eta_{CP} \frac{V_{td}V_{tb}^{*}}{V_{td}^{*}V_{tb}} \frac{V_{cb}V_{cd}^{*}}{V_{cb}^{*}V_{cd}} \]

\[ e^{-i2\beta} \]

\[ |\lambda_{CP}| = 1 \]

\[ \text{no possibility to generate this way direct or indirect CPV} \]

\[ \text{Im} \lambda_{CP} = -\eta_{CP} \sin 2\beta \]

\[ C_{fCP} = 0 \]

\[ S_{fCP} = -\eta_{CP} \sin 2\beta \]

\[ J/\psi (cc) \to J^{PC} = 1^{-} \]

\[ K_S \sim K_1 \to \eta_{CP} = +1 \]

\[ L=1 \to P = (-1)^L \]

\[ \eta_{CP} (J/\psi K_S) = -1 \]

\[ \eta_{CP} (J/\psi K_L) = +1 \]

CPV in interference between mixing and decay
Why $J/\psi K_{S,L}$ mode is golden

-leading-order tree decays to $c\bar{c}s$ final states

\[
\begin{align*}
\bar{b} & \quad \rightarrow \quad V_{cb}^* \quad \bar{c} \\
& \quad \downarrow \quad \downarrow \\
& \quad \quad \quad \quad \quad C \\
& \quad \quad \quad \quad \quad \downarrow \\
& \quad \quad \quad \quad \quad \bar{s} + d \quad K^0 \rightarrow K_{S,L} \\
& \quad \quad \quad \quad \quad \leftarrow \quad \leftarrow \\
& \quad \quad \quad \quad \quad V_{cs} \\
& \quad \quad \quad \quad \quad \downarrow \\
& \quad \quad \quad \quad \quad C
\end{align*}
\]

possible penguin contributions:

\[
\begin{align*}
\bar{b} & \quad \rightarrow \quad V_{xb}^* \quad \bar{s} \\
& \quad \downarrow \quad \downarrow \\
& \quad \quad \quad \quad \quad C \\
& \quad \quad \quad \quad \quad \downarrow \\
& \quad \quad \quad \quad \quad \bar{c} \\
& \quad \quad \quad \quad \quad \downarrow \\
& \quad \quad \quad \quad \quad C
\end{align*}
\]

where $x$ can be any up-type quark hence this counts for three penguin diagrams

can this be a problem?
Why $J/\psi K_{S,L}$ mode is golden

\[ \beta \equiv \text{arg} \left[ -V_{cd}V_{cb}^*/V_{td}V_{tb}^* \right] \]

\[
\begin{align*}
\bar{b} & \xrightarrow{V_{xb}^*} V_{xs} & \bar{s} \\
\bar{s} & \xrightarrow{V_{xs}} \bar{c} & \bar{c} \\
\bar{c} & \xrightarrow{V_{tb}} \bar{t} & \bar{t} \\
\text{using this unitary condition (2nd \leftrightarrow 3rd family), we eliminate } V_{tb}V_{ts}^* & \\
V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* = 0 & \rightarrow V_{tb}V_{ts}^* = -V_{ub}V_{us}^* - V_{cb}V_{cs}^* \\
\end{align*}
\]

thus the amplitude is:

\[
A_{ccs} \sim V_{cb}V_{cs}^* (T + P_c - P_t^*) + V_{ub}V_{us}^* (P_u - P_t^*)
\]

\( \mathcal{O}(\lambda^2) \) \hspace{1cm} \( \mathcal{O}(\lambda^4) \)

CKM-suppressed pollution by penguins
\[ \beta \equiv \arg \left[ -V_{cd}V_{cb}^*/V_{td}V_{tb}^* \right] \]

\[ \sin 2\beta \text{ in golden } b \rightarrow \bar{c}c\bar{s} \text{ modes} \]

\( \circ \) branching fraction: \( O(10^{-3}) \)
the colour-suppressed tree dominates
and the penguin pollution has
the same weak phase of the tree or is CKM suppressed

\[ A_{CP}(t) = \frac{\Gamma(\bar{B}^0(t) \rightarrow f_{CP}) - \Gamma(B^0(t) \rightarrow f_{CP})}{\Gamma(\bar{B}^0(t) \rightarrow f_{CP}) + \Gamma(B^0(t) \rightarrow f_{CP})} \]

\( \circ \) theoretical uncertainty:

\( \circ \) model-independent data-driven estimation from \( J/\psi\pi^0 \) data:
\[ \Delta S_{J/\psi K^0} = S_{J/\psi K^0} - \sin 2\beta = -0.01 \pm 0.01 \]

\( \circ \) model-dependent estimates of the u- and c- penguin biases
\[ \Delta S_{J/\psi K^0} = S_{J/\psi K^0} - \sin 2\beta \sim O(10^{-3}) \]
\[ \Delta S_{J/\psi K^0} = S_{J/\psi K^0} - \sin 2\beta \sim O(10^{-4}) \]

M. Bona – Flavour Physics and CP Violation – lecture 3
\[ \sin 2\beta \text{ in golden } b \rightarrow \bar{c}c_s \text{ modes} \]

Sine term has a non-zero coefficient and this tells us that there is CP violation in the interference between mixing and decay amplitudes in $\bar{c}c_s$ decays.

Heavy FLavour AVeraging (HFLAV) group for $J/\psi K_{S,L}$

\[ S = \sin 2\beta = 0.690 \pm 0.018 \]
\[ \beta \equiv \arg \left[ -V_{cd} V_{cb}^* / V_{td} V_{tb}^* \right] \]

\[ \sin 2\beta \text{ in } b \rightarrow c\bar{c}s \text{ modes} \]

\[ \sin(2\beta) \equiv \sin(2\phi_1) \]

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Reference</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>BaBar</td>
<td>PRD 79 (2009) 072009</td>
<td>$0.69 \pm 0.03 \pm 0.01$</td>
</tr>
<tr>
<td>BaBar $\chi_{K_S}$</td>
<td>PRD 80 (2009) 112001</td>
<td>$0.69 \pm 0.52 \pm 0.04 \pm 0.07$</td>
</tr>
<tr>
<td>BaBar $J/\psi$ (hadronic) $K_S$</td>
<td>PRD 69 (2004) 052001</td>
<td>$1.56 \pm 0.42 \pm 0.21$</td>
</tr>
<tr>
<td>Belle</td>
<td>PRL 103 (2012) 171802</td>
<td>$0.67 \pm 0.02 \pm 0.01$</td>
</tr>
<tr>
<td>ALEPH</td>
<td>PLB 492, 259 (2000)</td>
<td>$0.84^{+0.82}_{-1.04} \pm 0.16$</td>
</tr>
<tr>
<td>OPAL</td>
<td>EPJ C5, 379 (1998)</td>
<td>$3.20^{+1.80}_{-2.00} \pm 0.50$</td>
</tr>
<tr>
<td>CDF</td>
<td>PRD 61, 072005 (2000)</td>
<td>$0.79^{+0.41}_{-0.44}$</td>
</tr>
<tr>
<td>LHCb</td>
<td>JHEP 11 (2017) 170</td>
<td>$0.76 \pm 0.03$</td>
</tr>
<tr>
<td>Belle5S</td>
<td>PRL 108 (2012) 171801</td>
<td>$0.57 \pm 0.58 \pm 0.06$</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>$0.70 \pm 0.02$</td>
</tr>
</tbody>
</table>
\[ \beta \equiv \text{arg} \left[ -V_{cd}V_{cb}^*/V_{td}V_{tb}^* \right] \]

angle \( \beta \) in \( b \to c\overline{c}s \) modes
Searching for new physics via other $b \to c\bar{c}s$ modes

- $\sin^2\beta$ has been measured to $O(1^\circ)$ accuracy in $b \to c\bar{c}s$ decays.

- Can use this to search for signs of New Physics (NP) if:
  - Identify a rare decay sensitive to $\sin^2\beta$ (loop dominated process).
  - Measure $S$ precisely in that mode ($S_{\text{eff}}$).
  - Control the theoretical uncertainty on the Standard Model ‘pollution’ ($\Delta S_{\text{SM}}$).
  - Compute $\Delta S_{NP} = S_{\text{eff}} - S_{c\bar{c}s} - \Delta S_{\text{SM}}$.

- In the presence of NP: $\Delta S_{NP} \neq 0$

  - New heavy particles can introduce new amplitudes affecting physical observables of loop dominated processes.
  - Observables affected include branching fractions, CP asymmetries, forward backward asymmetries.. etc..
  - The Standard Model contributions need to be understood.
$\alpha \equiv \arg \left[ -V_{td}V_{tb}^* / V_{ud}V_{ub}^* \right]$

$b \rightarrow u u d$ transitions with possible loop contributions. Extract $\alpha$ using

- SU(2) Isospin relations.
- SU(3) flavour related processes.
\[ \alpha \equiv \arg[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*] \]

**CP violation: \( \alpha \)**

- Interference between box and tree results in an asymmetry that is sensitive to \( \alpha \) in \( B \rightarrow hh \) decays: \( h = \pi, \rho, \ldots \).

\[ C_{hh} = 0 \]
\[ S_{hh} = \sin(2\alpha) \]

This is again a case of interference between mixing and decay. This scenario is equivalent to the measurement of \( \sin 2\beta \) in Charmonium decays ... but in this case it is more complicated.
Interference between box and tree results in an asymmetry that is sensitive to $\alpha$ in $B \rightarrow hh$ decays: $h = \pi, \rho, \ldots$

In this case the penguin diagram is not CKM suppressed so it spoils the clean measurement of the CP violation effect.
\[ \alpha \equiv \arg \left[ -V_{td}V_{tb}^{*}/V_{ud}V_{ub}^{*} \right] \]

**CP violation: \( \alpha \)**

- Interference between box and tree results in an asymmetry that is sensitive to \( \alpha \) in \( B \rightarrow hh \) decays: \( h = \pi, \rho, \ldots \)

\[ C_{hh} = 0 \]
\[ S_{hh} = \sin(2\alpha) \]

- Measure \( S \propto \alpha_{\text{eff}} \)
- Need to determine \( \delta_{\alpha} = \alpha_{\text{eff}} - \alpha \) [P/T is different for each final state]

\[ C_{hh} \propto \sin(\delta) \]
\[ S_{hh} = \sqrt{1 - C_{hh}^2 \sin(2\alpha_{\text{eff}})} \]
\[ \delta = \delta_{P} - \delta_{T} \]
Several recipes describe how to bound penguins and measure $\alpha$.

- These are based on SU(2) [or SU(3)] symmetry.

**Bounding penguins**

- Use charged and neutral B decays to the $hh$ final state to constrain the penguin contribution and measure $\alpha$.

M. Gronau and D. London, 65, 3381 (1990)

- Use charged and neutral B decays to the $\rho \pi$ final state to constrain the penguin contribution and measure $\alpha$. Remove any overlapping regions in the Dalitz plot.


- Regions of the Dalitz plot with intersecting $\rho$ bands are included in this analysis; this helps resolve ambiguities.


\[ \alpha \equiv \arg \left( -V_{td} V_{tb}^* / V_{ud} V_{ub}^* \right) \]
\[ \alpha \equiv \text{arg}\left[ -V_{td}V_{tb}^*/V_{ud}V_{ub}^* \right] \]

**Isospin analysis**

\( \alpha \): collecting the ingredients

**from \( \alpha_{\text{eff}} \) → to \( \alpha \): isospin analysis**

○ \( B \rightarrow \pi^+\pi^- \), \( \pi^+\pi^0 \), \( \pi^0\pi^0 \) decays are connected from isospin relations

○ \( \pi\pi \) states can have \( I = 2 \) or \( I = 0 \)
  - the gluonic penguins contribute only to the \( I = 0 \) state (\( \Delta I = 1/2 \))
  - \( \pi^+\pi^0 \) is a pure \( I = 2 \) state (\( \Delta I = 3/2 \)) and it gets contribution only from the tree diagram

⇒ triangular relations allow for the determination of the phase difference induced on \( \alpha \):

Both \( \text{BR}(B^0) \) and \( \text{BR}(\bar{B}^0) \) have to be measured in all the \( \pi\pi \) channels

**Table:**

<table>
<thead>
<tr>
<th>Channel</th>
<th>Decay Amplitudes</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi\pi )</td>
<td>( A(B^+ \rightarrow \pi^+\pi^0) = \sqrt{3}/2 A_{3/2,2} )</td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{\sqrt{2}} A(B^0 \rightarrow \pi^+\pi^-) = \frac{1}{\sqrt{12}} A_{3/2,2} - \sqrt{\frac{1}{6}} A_{1/2,0} )</td>
</tr>
<tr>
<td></td>
<td>( A(B^0 \rightarrow \pi^0\pi^0) = \frac{1}{\sqrt{3}} A_{3/2,2} + \sqrt{\frac{1}{6}} A_{1/2,0} )</td>
</tr>
</tbody>
</table>
Consider the simplest case:

\[ B \rightarrow \pi\pi / \rho \rho \text{ decays.} \]

\[
\frac{1}{\sqrt{2}} A^{+-} + A^{00} = A^{+0}
\]

\[
\frac{1}{\sqrt{2}} \bar{A}^{+-} + \bar{A}^{00} = \bar{A}^{+0}
\]

There are SU(2) violating corrections to consider, for example electroweak penguins (~5%), but these are much smaller than current experimental accuracy and eventually they can be incorporated into the Isospin analysis.

\[ \delta \alpha = \alpha_{\text{eff}} - \alpha \]

Measuring S in h^0h^0 provides an additional constraint on this angle.
B → ππ

◎ Easy to isolate signal for π⁺π⁻ and π⁺π⁰ as these modes are relatively clean and have relatively large B ~ O(5 × 10⁻⁶).

◎ Much harder to isolate π⁰π⁰: B ~ 1.5 × 10⁻⁶
  ◊ No tracks in the final state to provide vertex info.
  ◊ B⁰ → π⁰π⁰ → γγγγ has a large ΔE resolution.

▷ Possible to separate flavour tags to measure C⁰⁰. This information completes the set of information required for an Isospin analysis.
\[ \alpha = \arg[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*] \]

\[ B \to \pi\pi \]
Isospin-related $\pi\pi$ decays

Simultaneous ML fit to all hh modes with h being $\pi$ or K:

- $B^+ \rightarrow \pi^+\pi^-, K^+\pi^-, K^+K^-$ (and cc)
- $B^+ \rightarrow \pi^+\pi^0, K^+\pi^0$ (and cc)
\( \alpha \equiv \arg \left[ -V_{td}V_{tb}^*/V_{ud}V_{ub}^* \right] \)

\[ B \rightarrow \pi\pi \]

- Inputs from:
  - \( B^0 \rightarrow \pi^+\pi^- \)
  - \( B^+ \rightarrow \pi^+\pi^0 \)
  - \( B^0 \rightarrow \pi^0\pi^0 \)

Additional information can be used: to reduce the degeneracy of the solutions and also to keep the amplitudes to go to infinity (unphysical). For example, \( B_s \rightarrow K\bar{K} \) (assuming SU(3) and a big uncertainty on that) can put an upper limit on the penguin amplitude.

Eight solutions to the isospin system: shown here a case with uncertainties reduced of a factor 10.

\[ \alpha = (90.1 \pm 2.2)^\circ \] (UTfit prediction)

\[ \alpha = (93.3 \pm 5.6)^\circ \] (combined SM)

UTfit prediction:

- \( \alpha = (90.1 \pm 2.2)^\circ \)
- \( \alpha = (93.3 \pm 5.6)^\circ \)
Vector-Vector modes: angular analysis required to
determine the CP content. L=0,1,2 partial waves:
- longitudinal: CP-even state
- transverse: mixed CP states
- +$: two $\pi^0$ in the final state
- wide $\rho$ resonance

but
- BR 5 times larger with respect to $\pi\pi$
- penguin pollution smaller than in $\pi\pi$
- $\rho$ are almost 100% polarized:
  - almost a pure CP-even state
\[ \alpha \equiv \arg\left[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*\right] \]

\[ B \to \rho \pi \ (\pi^+\pi^-\pi^0 \text{ Dalitz Plot}) \]

- **dominant decay** $\rho \pi$ is not a CP eigenstate

- **5 amplitudes** need to be considered:
  - $B^0 \to \rho^+\pi^-, \rho^-\pi^+, \rho^0\pi^0$, and $B^+ \to \rho^+\pi^0, \rho^0\pi^+$
  - Isospin pentagon

- **or time-dependent dalitz analysis**: $\alpha$ extraction together with the strong phases exploiting the amplitude interference:
  - Interference at equal masses-squared give information on the strong phases between resonances
\[ \gamma \equiv \arg \left[ -V_{ud}V_{ub}^{*} / V_{cd}V_{cb}^{*} \right] \]

Extract \( \gamma \) using \( B \rightarrow D^{(*)}K^{(*)} \) final states using:

- **GLW:** Use CP eigenstates of \( D^{0} \).
- **ADS:** Interference between doubly suppressed decays.
- **GGSZ:** Use the Dalitz structure of \( D \rightarrow K_{s}h^{+}h^{-} \) decays.

Measurements using neutral D mesons ignore D mixing.
$\gamma \equiv \arg[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*]$

\section*{$\gamma$ and DK trees}

- D(*)K(*) decays: from BRs and BR ratios, no time-dependent analysis, just rates
- the phase $\gamma$ is measured exploiting interferences: two amplitudes leading to the same final states
- some rates can be really small: $\sim 10^{-7}$

\begin{itemize}
  \item $V_{ub} = |V_{ub}|e^{-i\gamma}$ ($\sim \lambda^3$)
  \item $V_{cb} (\sim \lambda^2)$
\end{itemize}

Theoretically clean (no penguins neglecting the $D^0$ mixing)
Sensitivity to $\gamma$: the ratio $r_B$

\[ \gamma \equiv \arg[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*] \]

\[ V_{ub} = |V_{ub}|e^{-i\gamma} (\sim \lambda^3) \]

\[ \delta_B = \text{strong phase diff.} \]

$V_{cb} (\sim \lambda^2)$

\[ A(B^- \to D^0 K^-) = A_B \]
\[ A(B^+ \to \bar{D}^0 K^+) = A_B \]

\[ r_B = \frac{B^- \to \bar{D}^0 K^-}{B^- \to D^0 K^-} \]

\[ \approx 0.36 \quad \text{hadronic contribution} \]

\[ \text{channel-dependent} \]

\[ \text{to be measured: } r_B(DK), r_B^*(D^*K) \text{ and } r_B^s(DK^*) \]

in $B^+ \to D^{(*)0} K^+$: $r_B$ is $\sim 0.1$
Three ways to make DK interfere

GLW (Gronau, London, Wyler) method: uses the CP eigenstates $D^{(*)0}_{CP}$ with final states:
$K^+K^-$, $\pi^+\pi^-$ (CP-even), $K_s\pi^0(\omega,\phi)$ (CP-odd)

$$ R_{CP\pm} = 1 + r_B^2 \pm 2r_B \cos \gamma \cos \delta_B \quad A_{CP\pm} = \frac{\pm 2r_B \sin \gamma \sin \delta_B}{1 + r_B^2 \pm 2r_B \cos \gamma \cos \delta_B} $$

ADS (Atwood, Dunietz, Soni) method: $B^0$ and $\bar{B}^0$ in the same final state with $D^0 \rightarrow K^+\pi^-$ (suppr.) and $\bar{D}^0 \rightarrow K^+\pi^-$ (fav.)

$$ R_{ADS} = r_B^2 + r_{DCS}^2 + 2r_B r_{DCS} \cos \gamma \cos(\delta_B + \delta_D) $$

the most sensitive way to $\gamma$

$D^0$ Dalitz plot with the decays $B^- \rightarrow D^{(*)0}[K_S\pi^+\pi^-] K^-$

three free parameters to extract: $\gamma$, $r_B$ and $\delta_B$
GLW Method: Study $B^+ \rightarrow D_{CP}^0 X^+$ and $B^+ \rightarrow \bar{D} X^+$ cc ($\bar{D}^0 \rightarrow K^+\pi^-$)

- $X^+$ is a strangeness one meson e.g. a $K^+$ or $K^{*+}$.
- $D_{CP}^0$ is a CP eigenstate (use these to extract $\gamma$):

$$D_{CP=+1}^0 = K^+K^-\pi^+\pi^-$$

$$D_{CP=-1}^0 = K_S^0\pi^0, K_S^0\omega, K_S^0\phi$$

- $4$ observables
- $3$ unknowns: $r_B$, $\gamma$, and $\delta$

$$R_{CP\pm} = \frac{BF(B^- \rightarrow D_{\pm}^0 K^-) + BF(B^+ \rightarrow D_{\pm}^0 K^+)}{BF(B^- \rightarrow D^0 K^-) + BF(B^+ \rightarrow D^0 K^+)} = 1 + r_B^2 \pm 2 r_B \cos \delta \cos \gamma$$

$$A_{CP\pm} = \frac{BF(B^- \rightarrow D_{\pm}^0 K^-) - BF(B^+ \rightarrow D_{\pm}^0 K^+)}{BF(B^- \rightarrow D_{\pm}^0 K^-) + BF(B^+ \rightarrow D_{\pm}^0 K^+)} = \pm 2 r_B \sin \delta \sin \gamma / R_{CP\pm}$$

- The precision on $\gamma$ is strongly dependent on the value of $r_B$.
  - $r_B \sim 0.1$ as this is a ratio of Cabibbo suppressed to Cabibbo allowed decays and also includes a colour suppression factor for $B^+ \rightarrow D^{(*)}K^{(*)}$ $b \rightarrow u$ decays.
  - Measurement has an $8$-fold ambiguity on $\gamma$.

\[ \gamma \equiv \arg \left[ -V_{ud}V_{ub}^*/V_{cd}V_{cb}^* \right] \]

\[ \gamma: \text{ADS Method} \]

- **ADS Method: Study** \( B^{\pm,0} \rightarrow D^{(*)0} K^{(*)\pm} \)

- Reconstruct doubly suppressed decays with common final states and extract \( \gamma \) through interference between these amplitudes:

  \( B^- \rightarrow D^{(*)0} K^{(*)-} \)

  CKM Favoured

  \( D^{(*)0} \rightarrow K^+ \pi^- \)

  Doubly CKM Suppressed

\[ B^- \rightarrow \bar{D}^{(*)0} K^{(*)-} \]

CKM and Color Suppressed

\[ \bar{D}^{(*)0} \rightarrow K^+ \pi^- \]

CKM Favoured

- \( \gamma \) extracted using ratios of rates:

  \( r_B^{(*)} = \left| \frac{A(B^- \rightarrow \bar{D}^{(*)0} K^-)}{A(B^- \rightarrow D^{(*)0} K^-)} \right| \)

    \( \delta^{(*)} = \delta^{(*)}_B + \delta_D \)

    \( \delta^{(*)} \) is the sum of strong phase differences between the two \( B \) and \( D \) decay amplitudes.

  \( r_D = \left| \frac{A(D^0 \rightarrow K^+ \pi^-)}{A(D^0 \rightarrow K^- \pi^+)} \right| \)

    \( r_D \) and \( r_B \) are measured in \( B \) and charm factories.

    \( \delta_D \) is measured by CLEO-c
\[ \gamma \equiv \arg[-V_{ud}V^*_{ub}/V_{cd}V^*_{cb}] \]

\section*{GGSZ Method}

- **GGSZ ("Dalitz") Method**: Study \( D^{(*)0}K^{(*)} \) using the \( D^{(*)0} \rightarrow K_s h^+ h^- \) Dalitz structure to constrain \( \gamma \). (\( h = \pi, K \))
  - Self tagging: use charge for \( B^\pm \) decays or \( K^{(*)} \) flavour for \( B^0 \) mesons.
    \[
    A(B^\pm \rightarrow (K^0_S h^+ h^-)_D K^\pm) \propto f(m_+, m_-) + f(m_-, m_+) r_B e^{i(\delta_B \pm \gamma)}
    \]
    where \( m_\pm = m_{K^0_S h^\pm} \)
  - Need detailed model of the amplitudes in the D meson Dalitz plot.
  - Use a control sample (CLEO-c data or \( D^{*+} \rightarrow D^0 \pi^+ \)) to measure the Dalitz plot.

\[
D^{*+} \rightarrow D^0 \pi^-
\]
\[
\downarrow
\]
\[
D^0 \rightarrow K^0_S h^+ h^-
\]

Control sample plots from BaBar GGSZ paper
\[ \gamma \equiv \arg \left[ -V_{ud}V_{ub}^*/V_{cd}V_{cb}^* \right] \]

\[ \gamma : \text{GGSZ Method} \]

○ neutral D mesons reconstructed in three-body CP-eigenstate final states
  (typically \( D^0 \rightarrow K_S\pi^-\pi^+ \))

○ the complete structure (amplitude and strong phases) of the \( D^0 \) decay in the phase space is obtained on independent data sets and used as input to the analysis

○ use of the cartesian coordinate:
  ○ \( x_\pm = r_B \cos (\delta \pm \gamma) \)
  ○ \( y_\pm = r_B \sin (\delta \pm \gamma) \)

○ \( \gamma \), \( r_B \) and \( \delta_B \) are obtained from a simultaneous fit of the \( K_S\pi^+\pi^- \) Dalitz plot density for \( B^+ \) and \( B^- \)

○ need a model for the Dalitz amplitudes

○ 2-fold ambiguity on \( \gamma \)
CP violation: $\gamma$

$\gamma \equiv \arg[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*]$

$\gamma$ from B into DK decays:

combined: $(73.4 \pm 4.4)^\circ$

UTfit prediction: $(65.8 \pm 2.2)^\circ$
back-up
CP violation in interference between mixing and decay:

◎ decays in final state $f$
  accessible to both a $B$ or a $\bar{B}$
  ($f$ is not necessarily a CP eigenstate)

◎ if $\text{Im} \lambda \neq 0$ then $\rightarrow$ CP violation

\[
\lambda_{f_{\text{CP}}} = \frac{q}{p} \cdot \frac{\bar{A}_{f_{\text{CP}}}}{A_{f_{\text{CP}}}}
\]

\[
\lambda = \frac{q A(\bar{B} \to f)}{p A(B \to f)} = \frac{V_{td}^* V_{tb} \bar{A}}{V_{td} V_{tb}^* A} \sim e^{-i2\beta} \frac{\bar{A}}{A}
\]

**examples**

| mixing        | $f$                                      | $\text{Arg}(\frac{A}{A})$ | $|\lambda|$ | $\text{parameter}$ |
|---------------|------------------------------------------|----------------------------|--------------|---------------------|
| "sin $2\beta$" | $B^0 \to l\nu X, D^{(*)}\pi(\rho, \alpha_1)$ | 0                          | $\sim 0$    | $\Delta M_{B^0}$    |
| "sin $2\alpha$" | $B^0 \to J/\psi K^0, \ldots$            | 0                          | 1            | $\sin 2\beta$      |
| "sin $(2\beta + \gamma)$" | $B^0 \to \pi\pi, \rho\pi, \pi\pi\pi$ | $\sim (-2\gamma)$        | $\sim 1$    | $\sin 2\alpha$     |
|               | $B^0 \to D^{(*)}\pi$                    | $\sim (-\gamma)$          | $\sim 0.02$ | $\sin(2\beta + \gamma)$ |
The $B^0$ and $\bar{B}^0$ mesons from the $Y(4S)$ are in a coherent $L = 1$ state:

- The $Y(4S)$ is a $b\bar{b}$ state with $J^{PC} = 1^{--}$.
- $B$ mesons are scalars ($J^P = 0^{-}$)
  - total angular momentum conservation
  - the $B\bar{B}$ pair has to be produced in a $L = 1$ state.

The $Y(4S)$ decays strongly so $B$ mesons are produced in the two flavour eigenstates $B^0$ and $\bar{B}^0$:

- After production, each $B$ evolves in time, but in phase so that at any time there is always exactly one $B^0$ and one $\bar{B}^0$ present, at least until one particle decays:
  - If at a given time $t$ one $B$ could oscillate independently from the other, they could become a state made up of two identical mesons: but the $L = 1$ state is anti-symmetric, while a system of two identical mesons (bosons!) must be completely symmetric for the two particle exchange.

Once one $B$ decays the other continues to evolve, and so it is possible to have events with two $B$ or two $\bar{B}$ decays.
Asymmetric energies produce boosted $\Upsilon(4S)$, decaying into coherent $B \bar{B}$ pair.

Fully reconstruct decay to state or admixture under study ($B_{RECO}$)

Measuring $\Delta t$
Measuring $\Delta t$

Asymmetric energies produce boosted $Y(4S)$, decaying into coherent $B\bar{B}$ pair

- $\beta\gamma = 0.56$ (BaBar)
  \quad = 0.425$ (Belle)
- $t = t_1$ corresponds to the time that $B_{\text{TAG}}$ decays.
- $t_2 - t_1 = \Delta t$

$\Delta z = (\beta\gamma c) \Delta t$

Determine time between decays from vertices

Fully reconstruct decay to state or admixture under study ($B_{\text{RECO}}$)
Measuring $\Delta t$

Asymmetric energies produce boosted $\Upsilon(4S)$, decaying into coherent $B\bar{B}$ pair

- $\beta \gamma = 0.56$ (BaBar)
  $= 0.425$ (Belle)

- $t = t_1$ corresponds to the time that $B_{\text{TAG}}$ decays.

- $t_2 - t_1 = \Delta t$

$\Rightarrow$ Then fit the $\Delta t$ distribution to obtain the amplitude of sine and cosine terms.
\[ \alpha \equiv \arg \left[ -V_{td}V_{tb}^{*}/V_{ud}V_{ub}^{*} \right] \]

\[ \mathbf{B} \rightarrow \rho\rho \]

- (simplified) angular analysis

- Inputs from:
  
  \[ \mathbf{B}^{0} \rightarrow \rho^{+}\rho^{-} \]
  
  \[ \mathbf{B}^{+} \rightarrow \rho^{+}\rho^{0} \]
  
  \[ \mathbf{B}^{0} \rightarrow \rho^{0}\rho^{0} \]

\( \theta_{i} \) are the helicity angles: angles between the \( \pi^{0} \) momentum and the direction opposite to that of the \( \mathbf{B}^{0} \) in the vector rest frame.

\( \phi \) is the angle between the vector meson decay planes.

- We define the fraction of longitudinally polarised events as:

\[
\frac{\Gamma_{L}}{\Gamma} = \frac{|H_{0}|^{2}}{|H_{0}|^{2} + |H_{+1}|^{2} + |H_{-1}|^{2}},
\]

\[
\frac{d^{2}\Gamma}{\Gamma d \cos \theta_{1} d \cos \theta_{2}} = \frac{9}{4} \left[ f_{L} \cos^{2} \theta_{1} \cos^{2} \theta_{2} + \frac{1}{4} (1 - f_{L}) \sin^{2} \theta_{1} \sin^{2} \theta_{2} \right]
\]

- \( f_{L} \sim 1 \) for \( \mathbf{B} \rightarrow \rho\rho \) decays: this helps simplify extracting \( \alpha \).

- Can measure \( S^{00} \) as well as \( C^{00} \) to help resolve ambiguities.

- Finite width of the \( \rho \) is ignored in the \( \alpha \) determination
CP violation: $\alpha$

Combing all the modes to maximize our knowledge of $\alpha$.

Bayesian analysis: the quantity plotted is now the Probability Density Function (PDF).
\[ \alpha \equiv \arg[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*] \]

\[ \mathbf{B} \to \rho\pi \ (\pi^+\pi^-\pi^0 \text{ Dalitz Plot}) \]

- Analyse a transformed Dalitz Plot to extract parameters related to \( \alpha \).
- Use the Snyder-Quinn method.

\[ \rho \pm \pi^\mp \]

\[ s_- = (p_+ + p_0)^2 \]

\[ s_+ = (p_+ + p_0)^2 \]

\[ \sqrt{s} = 1.5 \text{ GeV/c}^2 \]

\[ \sqrt{s_0} = 1.5 \text{ GeV/c}^2 \]

\[ m_0 = m_p + p_0 - p_0 \]

\[ \theta_0 \equiv \frac{1}{\pi} \theta \]

\[ \theta' = \theta_0 \]

\[ m' \equiv \frac{1}{\pi} \arccos \left( \frac{2m_0 - m_{0\min}}{m_{0\max} - m_{0\min}} - 1 \right) \]

\[ (m_0 = m_{\pi^+\pi^-}) \]

- Fit the time-dependence of the amplitudes in the Dalitz plot:

\[ |A_{3\pi}^\pm(\Delta t)|^2 = e^{-|\Delta t|/\tau_{B^0}} \left[ |A_{3\pi}|^2 + |\bar{A}_{3\pi}|^2 \mp \left( |A_{3\pi}|^2 - |\bar{A}_{3\pi}|^2 \right) \cos(\Delta m_d \Delta t) \right] \]

\[ \pm 2\text{Im} \left[ A_{3\pi} \bar{A}_{3\pi}^* \right] \sin(\Delta m_d \Delta t) \]