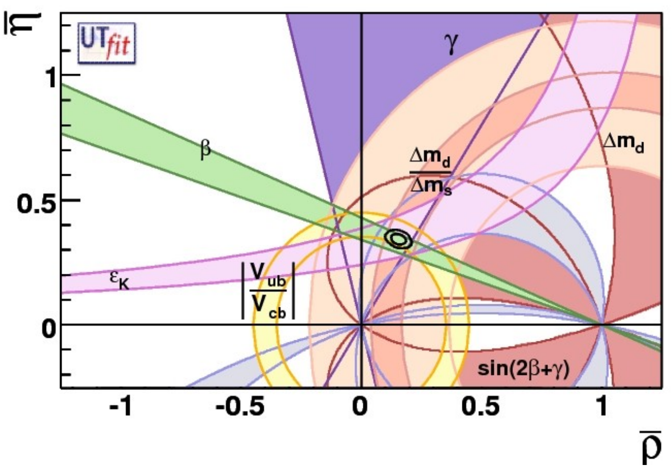
Flavour Physics and CP Violation



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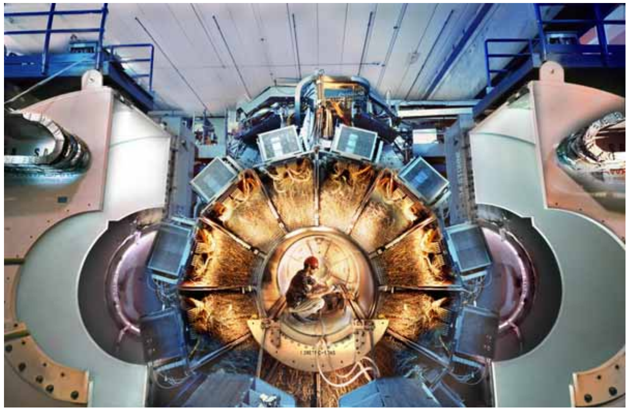


Post-FPCP 2018 Summer School

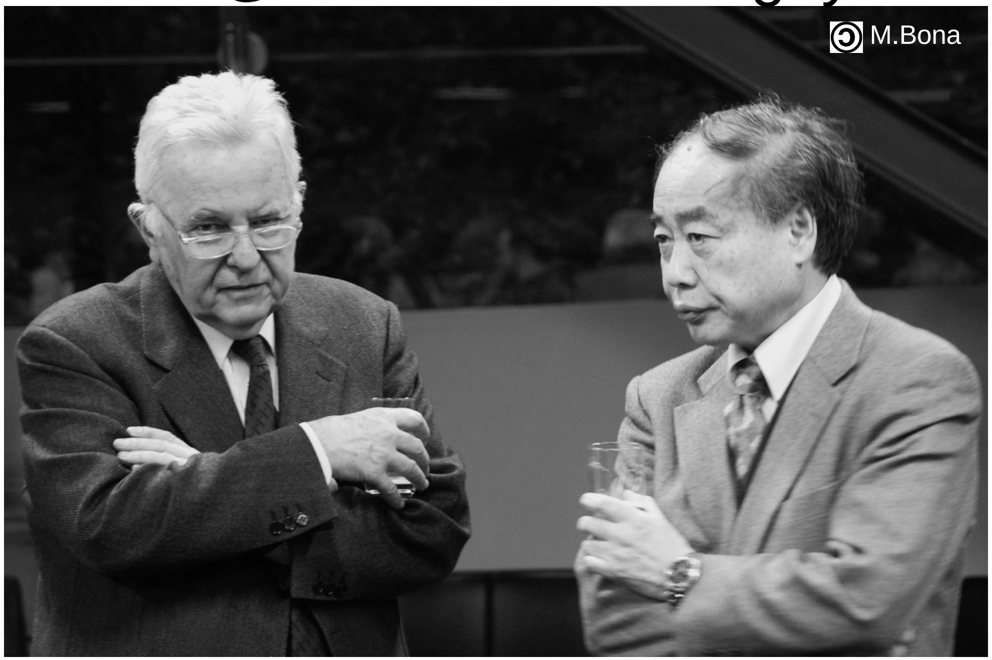
IIT Hyderabad, India

Lecture 2

Outline

- ◈ CKM matrix
- ◈ Unitarity Triangle
- ◈ types of CP violation
- ◈ angles of the unitarity triangle

CK @ CKM2006 in Nagoya



What causes the difference between matter and antimatter?

- The CKM matrix arises from the relative misalignment of the Yukawa matrices for the up- and down-type quarks:
  - It is a 3x3 complex unitary matrix described by 4 (real) parameters:
    - 3 can be expressed as (Euler) mixing angles
    - the fourth makes the CKM matrix complex (i.e. gives it a phase)
      - weak interaction couplings differ for quarks and antiquarks
      - CP violation

CKM matrix in the Standard Model

The charged current interactions get a flavour structure encoded in the Cabibbo-Kobayashi-Maskawa (CKM) matrix V:

$$\boxed{\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\left(\bar{\tilde{U}}_L \gamma^\mu W_\mu^+ V \tilde{D}_L + \bar{\tilde{D}}_L \gamma^\mu W_\mu^- V^\dagger \tilde{U}_L\right).}$$

V_{ij} connects left-handed up-type quark of the *i*th generation to left-handed down-type quark of *j*th generation. Intuitive labelling by flavour:

$$\boxed{V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}, \quad V_{13} = V_{ub} \;\; etc$$

Matrix V is unitary by construction

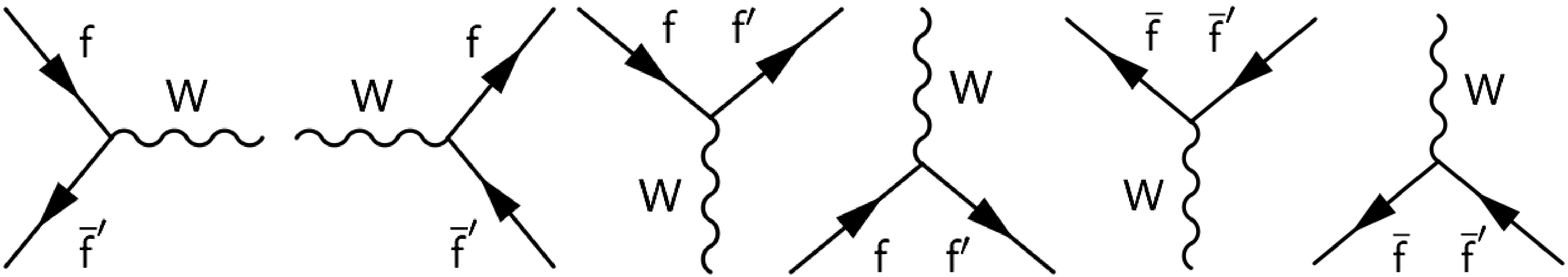
Via W exchange is the only way to change flavour in the SM.

CKM matrix in the Standard Model

Quarks change type in weak interactions.

We parameterise the couplings V_{ij} in the CKM matrix.

All the possible weak interaction involving a W are combinations of:



Where f = e, μ, τ, d, s, b

f' = ν_e, ν_μ, ν_τ, u, c, t

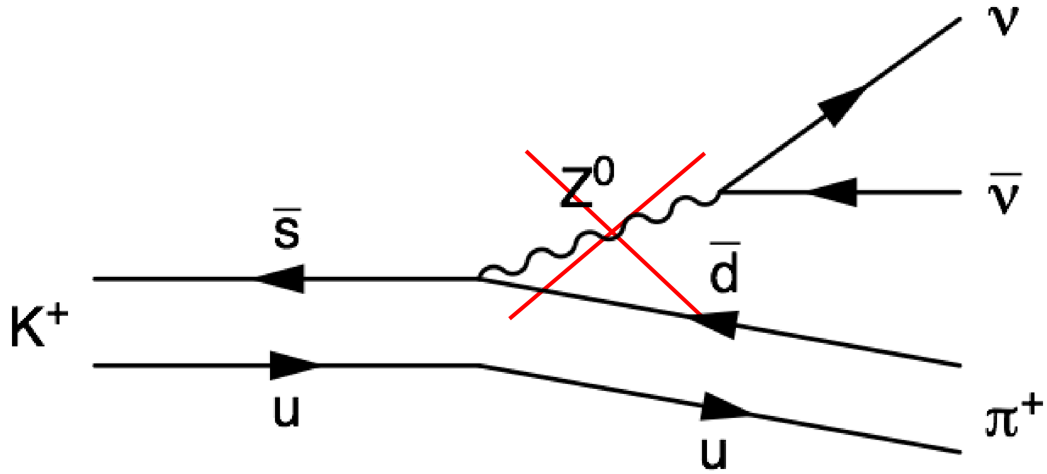
The $W^{\pm}$ is flavour changing i.e. u $\rightarrow$ d, s $\rightarrow$ u etc

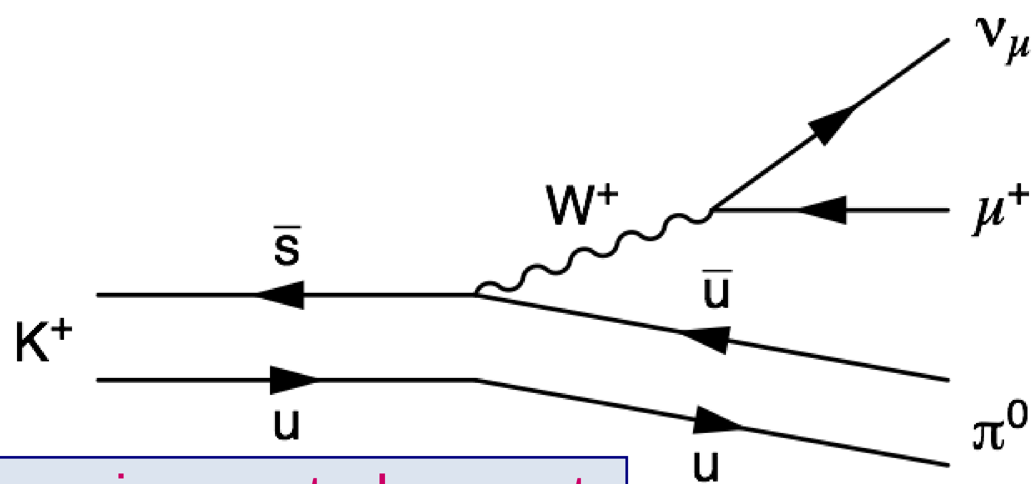
No Flavour Changing Neutral Currents

All observed neutral currents are found to obey $\Delta S = 0$

Measure ratio:

$$\frac{K^+ \to \pi^+ + \nu + \bar{\nu}}{K^+ \to \pi^0 + \mu^+ + \nu_\mu} = \ < 10^{-5}$$





There are no flavour changing neutral currents

CKM matrix in the Standard Model

With u, d, s, c quarks the weak charged current is given by:

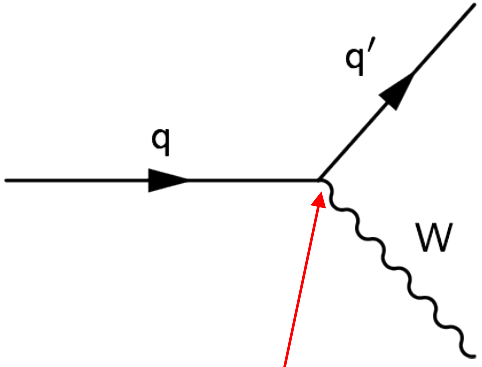
$$J^{-} = (\bar{u}, \bar{c}) \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Normal coordinate rotation matrix

$$\begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix}$$

With u, d, s, c, b, t quarks this becomes:

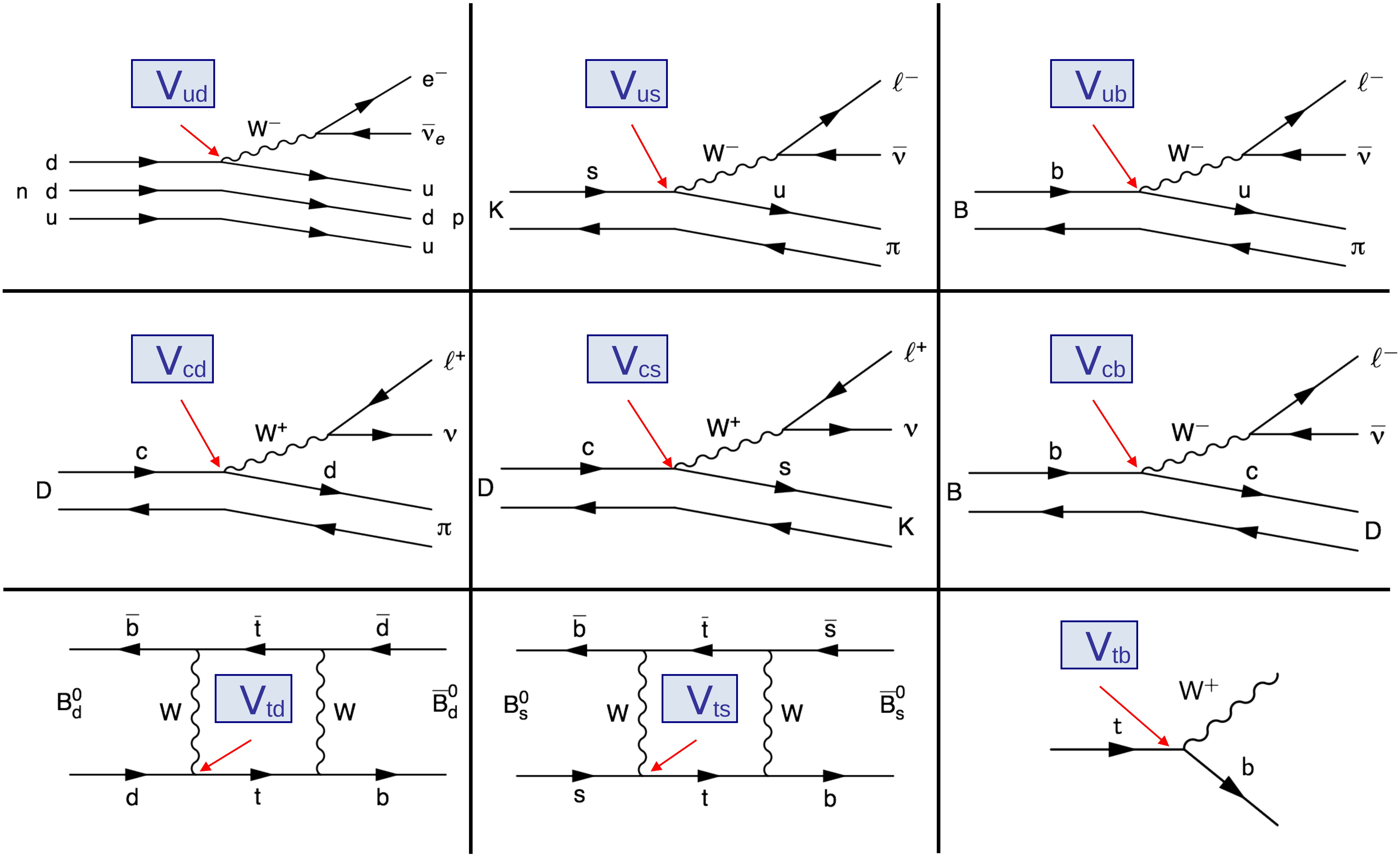
$$J^{-} = (\bar{u}, \bar{c}, \bar{t}) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



Put $V_{qq'}$ in amplitude

The Cabibbo, Kobayashi, Maskawa (CKM) Matrix

CKM matrix in the Standard Model



CKM matrix: rotation decomposition

The CKM matrix can be seen as the product of three rotation matrices and each rotation involves two of the three families:

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}e^{i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

which gives the classic exact parameterisation that can be found for example on the PDG:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

with $c_{ij}=\cos\theta_{ij}$ and $s_{ij}=\sin\theta_{ij}$, and i,j=1,2,3. δ is the CP violating phase

Unitary matrix independent parameters

in general, an n $\times$ n unitary matrix has n^2 real and independent parameters:

- a n $\times$ n matrix would have $2n^2$ parameters
- the unitary condition imposes n normalization constraints
- n(n - 1) conditions from the orthogonality between each pair of columns:

thus $2n^2 - n - n(n - 1) = n^2$.

In the CKM matrix, not all of these parameters have a physical meaning:

- given n quark generations, 2n - 1 phases can be absorbed by the freedom to select the phases of the quark fields
  - Each u, c or t phase allows for multiplying a row of the CKM matrix by a phase, while each d, s or b phase allows for multiplying a column by a phase.

thus: $n^2 - (2n - 1) = (n - 1)^2$.

Among the n^2 real independent parameters of a generic unitary matrix:

- ½ n(n - 1) of these parameters can be associated to real rotation angles,

so the number of independent phases in the CKM matrix case is:

$n^2 - \frac{1}{2} n(n-1) - (2n - 1) = \frac{1}{2}(n - 1)(n - 2)$

n(families)	Total indep. params. $(n-1)^2$	Real rot. angles $\frac{1}{2}n(n-1)$	Complex phase factors $\frac{1}{2}(n-1)(n-2)$
2	1	1	0
3	4	3	1
4	9	6	3

CKM matrix: Wolfenstein parameterisation

From measurements, V results hierarchical $\rightarrow$ $\theta_{13} \ll \theta_{23} \ll \theta_{12}$

We can see this hierarchy via the Wolfenstein parameterisation:

$\rightarrow$ the CKM matrix elements are expanded in order of sin θ_{12}

historically called Cabibbo angle θ_C:

$\rightarrow$ Wolfenstein parameter $\lambda = \sin\theta_{12} \sim 0.22$

$$V_{CKM} = \boxed{\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}} + \mathcal{O}(\lambda^4)$$

$\rightarrow$ Wolfenstein parameters: $\lambda \sim 0.22$, $A \sim 0.83$, $\rho \sim 0.15$, $\eta \sim 0.35$

CKM matrix: Wolfenstein parameterisation

From the Wolfenstein parameter $\lambda = \sin\theta_{12} \sim 0.22$, we can get an idea on the sizes of the various CKM matrix elements:

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At λ^2 order, the third generation decouples

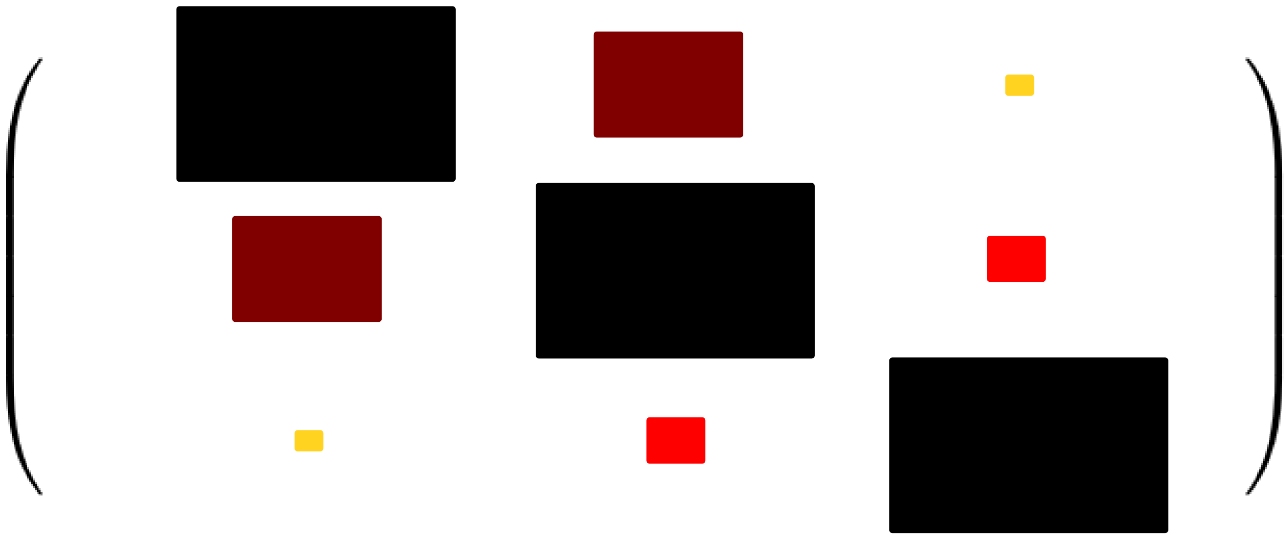
$\eta \neq 0$ signals CP violation

→ imaginary part of the V_{ub} and V_{td} elements (1st ⇄ 3rd family)

CKM matrix: Wolfenstein parameterisation

From the Wolfenstein parameter $\lambda = \sin\theta_{12} \sim 0.22$, we can get an idea on the sizes of the various CKM matrix elements:

$V_{CKM} =$



$+ \mathcal{O}(\lambda^4)$

So the preferred decays are $t \rightarrow b \rightarrow c \rightarrow s \rightarrow u$

CKM matrix: values of the elements

The CKM Matrix is approximately diagonal:

$$J^{-} \simeq \bar{u}d + \bar{c}s + \bar{t}b$$

$$\text{CKM} \simeq \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ -0.226 & 0.973 & 0.041 \\ 0.009 & -0.041 & 0.999 \end{pmatrix}$$

These numbers are obtained with their uncertainties from the processes mentioned before or global fits

$$V_{CKM} = \begin{pmatrix} (0.97431 \pm 0.00012) & (0.22514 \pm 0.00055) & (0.00365 \pm 0.00010)e^{i(-66.8\pm2.0)^\circ} \\ (-0.22500 \pm 0.00054)e^{i(0.0351\pm0.0010)^\circ} & (0.97344 \pm 0.00012)e^{i(-0.001880\pm0.000052)^\circ} & (0.04241 \pm 0.00065) \\ (0.00869 \pm 0.00014)e^{i(-22.23\pm0.63)^\circ} & (-0.04124 \pm 0.00056)e^{i(1.056\pm0.032)^\circ} & (0.999112 \pm 0.000024) \end{pmatrix}$$

summer 2018 analysis from UTfit: www.utfit.org

CKM matrix: Buras-Wolfenstein parameterisation

Usually the Buras correction to the Wolfenstein parameterisation is used: $\bar{\rho} = \rho\ (1- \lambda^2/2)$

$\bar{\eta} = \eta\ (1-\lambda^2/2)$

$$V = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\bar{\rho}-i\bar{\eta}) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Looks indentical to the Wolfenstein one but now the matrix is unitary also in this "approximation" at all λ orders.

Also $\bar{\rho}$ +i$\bar{\eta}$ is phase-convention independent:

$$\boxed{\bar{\rho} + i\bar{\eta} \equiv -\frac{V_{\mathrm{ud}} V^*_{\mathrm{ub}}}{V_{\mathrm{cd}} V^*_{\mathrm{cb}}}}$$

Unitarity relations

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

multiply with its hermitian conjugate
(complex conjugate + transpose)
$VV^{\dagger} = V^{\dagger}V = \mathbf{1}$

$$\Sigma_i V_{ij} V^*_{ik} = \delta_{jk}$$

column orthogonality

$$\Sigma_j V_{ij} V^*_{kj} = \delta_{ik}$$

row orthogonality

The six vanishing combinations can be represented as triangles in a complex plane

Unitarity relations

The triangles obtained by taking scalar products of neighboring rows or columns are nearly degenerate. However, the areas of all triangles are the same, half of the Jarlskog invariant J.

1st $\rightleftarrows$ 2nd family

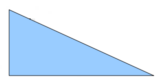
$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* \simeq \mathcal{O}(\lambda) + \mathcal{O}(\lambda) + \mathcal{O}(\lambda^5) = 0$$

2nd $\rightleftarrows$ 3rd family

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* \simeq \mathcal{O}(\lambda^4) + \mathcal{O}(\lambda^2) + \mathcal{O}(\lambda^2) = 0$$

triangles
not to scale

1st $\rightleftarrows$ 3rd family



$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* \simeq \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) = 0$$

Jarlskog invariant J

J ∝ determinant of the commutator of the mass matrices for the up-type quarks and the down-type quarks:

⇨ the commutator tells us if the two matrices can be simultaneously diagonalised or not

⇨ this specific determinant vanishes if and only if there are no CP violating terms

It is a phase-convention-independent measure of CP violation and it defined as:

$$\mathrm{Im}[V_{ij}V_{kl}V_{il}^{*}V_{kj}^{*}] = J\sum_{m,n}\varepsilon_{ikm}\varepsilon_{jln}$$

in the PDG parameterisation of the CKM matrix:

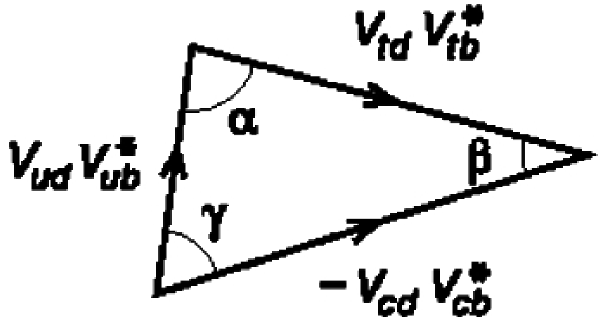
$$\mathbf{J = s_{12}s_{13}s_{23}c_{12}c_{13}^{2}c_{23}\,\sin\delta.}$$

This is twice the area of the unitarity triangles

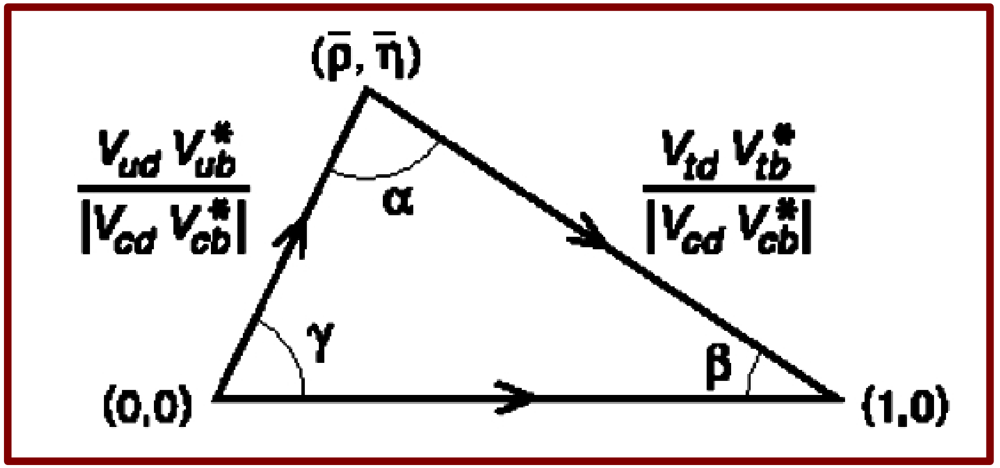
Third unitarity relation

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* \simeq \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) = 0$$

$V_{id}V^*_{ib} = 0$ represents the orthogonality condition between the first and the third column of the CKM matrix (the orientation depends on the phase convention)



re-scaled version where sides have been divided by $|V_{cd}V^*_{cb}|$

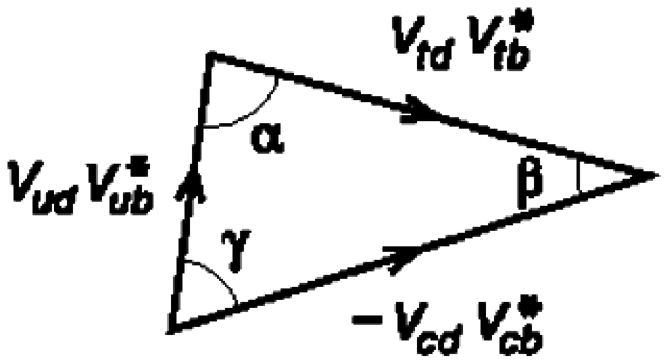


In terms of the Wolfenstein parameterization, the coordinates of this triangle are (0, 0), (1, 0) and $(\bar{\rho}, \bar{\eta})$:

the two sides are $(\bar{\rho} + i\bar{\eta})$ and $(1 - \bar{\rho} - i\bar{\eta})$.

The Unitarity Triangle

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* \simeq \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) = 0$$

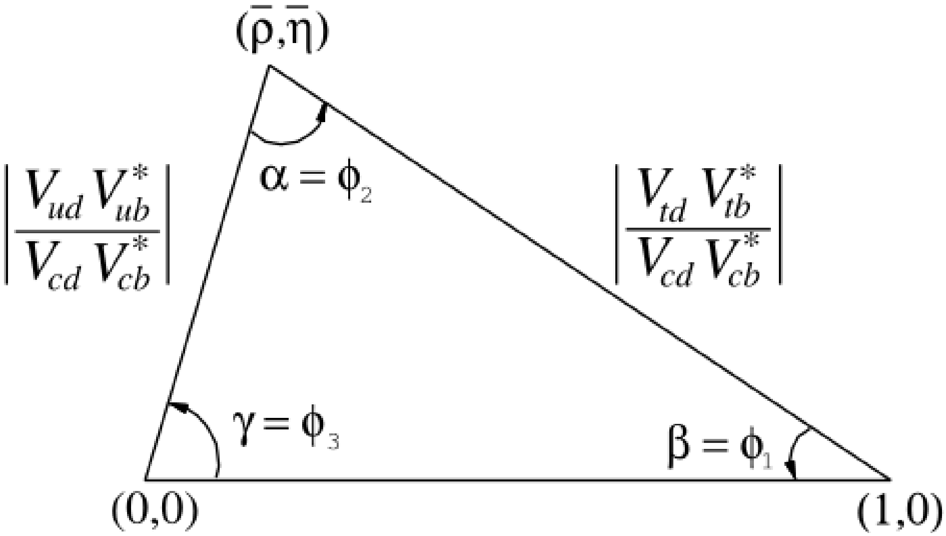


The angles can be written in terms of CKM matrix elements as:

$$\alpha \equiv \arg[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*]$$
$$\beta \equiv \arg[-V_{cd}V_{cb}^*/V_{td}V_{tb}^*]$$
$$\gamma \equiv \arg[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*]$$

The Unitarity Triangle

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* \simeq \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) = 0$$



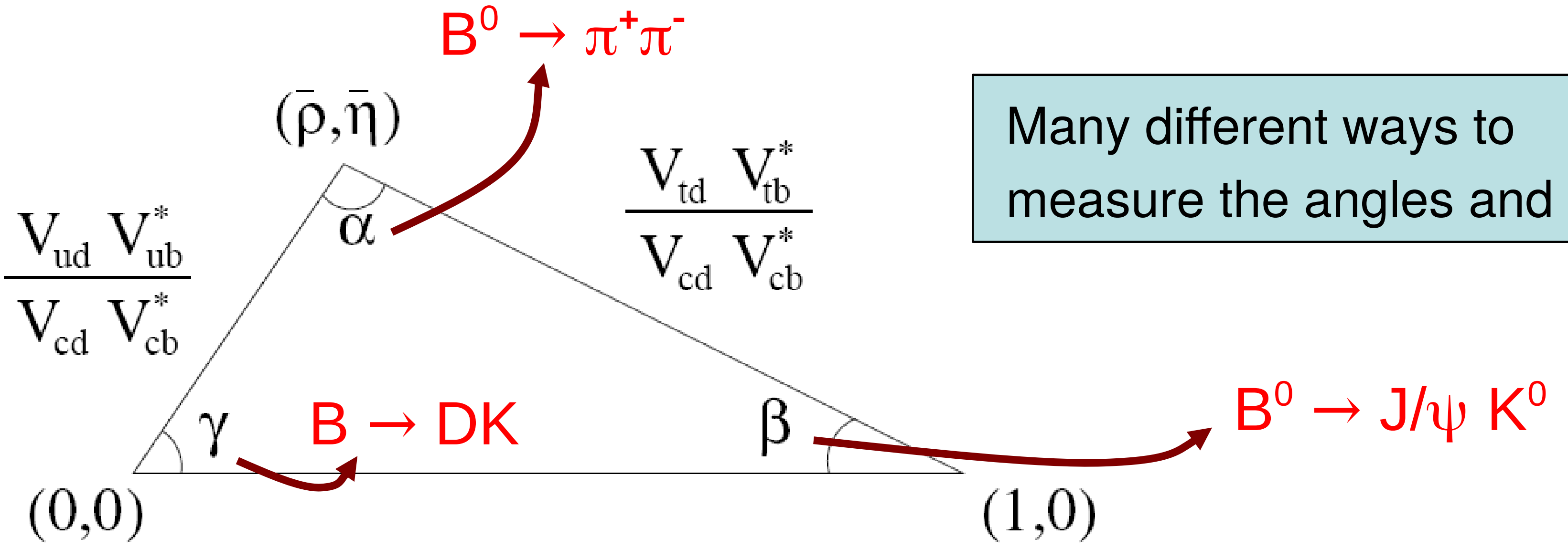
The angles can be written in terms of CKM matrix elements as:

$$\alpha \equiv \arg[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*]$$
$$\beta \equiv \arg[-V_{cd}V_{cb}^*/V_{td}V_{tb}^*]$$
$$\gamma \equiv \arg[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*]$$

In the Wolfenstein parameterisation:

- the β/ϕ_1 angle corresponds to the phase of V_{td}
- the γ/ϕ_3 angle corresponds to the phase of V_{ub}
- the α/ϕ_2 angle can be obtained with $\pi - \beta - \gamma$ (assumes unitarity)

Probing the structure of the CKM mechanism



Many different ways to measure the angles and sides.

- We need to measure the angles and sides to over-constrain this triangle, and test that it closes.
- Need to define observables and experiments to measure these quantities

The Unitarity Triangle

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

many observables functions of $\bar{\rho}$ and $\bar{\eta}$: overconstraining

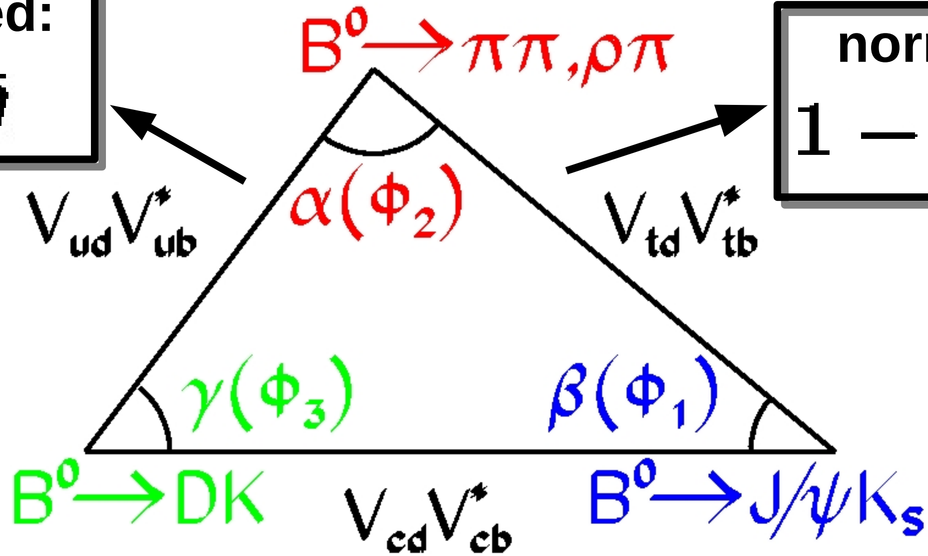
$$\alpha = \pi - \beta - \gamma$$

normalized:

$$\bar{\rho} + i\bar{\eta}$$

normalized:

$$1 - \bar{\rho} - i\bar{\eta}$$



$$\gamma = atan\left(\frac{\bar{\eta}}{\bar{\rho}}\right)$$

$$\beta = atan\left(\frac{\bar{\eta}}{(1-\bar{\rho})}\right)$$

Types of CP Violation in the SM

Three Types of CP Violation

Need more than one amplitude to have a non-zero CP violation: *interference*

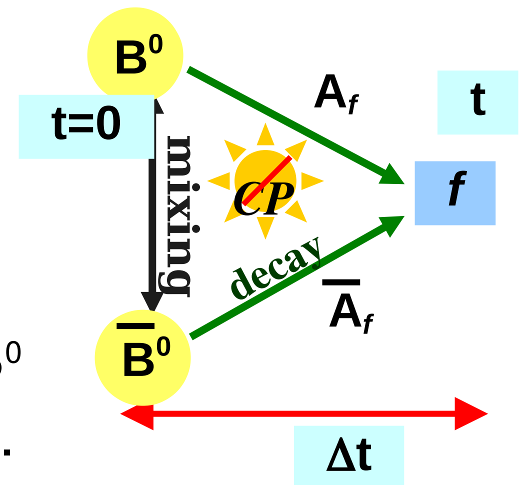
1. Indirect CP violation, or CPV in mixing:

$$P(B^0 \rightarrow \overline{B}^0) \neq P(\overline{B}^0 \rightarrow B^0)$$

2. Direct CP violation, or CPV in the decay:

$$P(B^0 \rightarrow f) \neq P(\overline{B}^0 \rightarrow \overline{f})$$

3. CPV in the interference between mixing and decay.



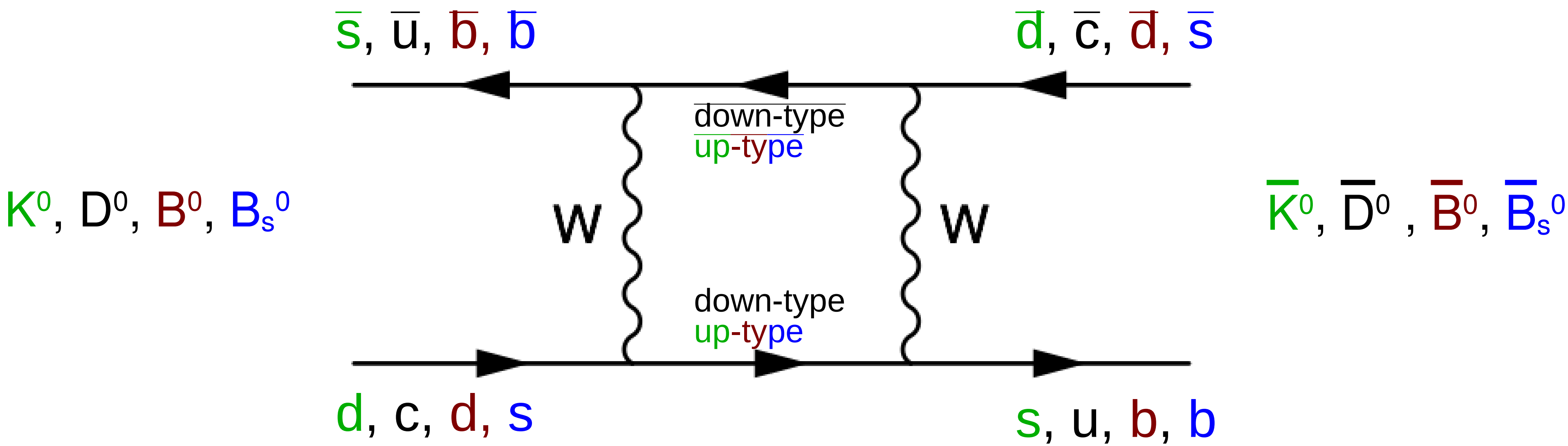
Cartoon shows the decay of a B^0 or $\overline{B}^0$ into a common final state f.

Aside: Neutral Meson Systems

The amazing case of neutral non-flavourless meson systems
$\rightarrow$ considering neutral mesons $u\bar{u}'$ where u has a different flavour with respect to u' $\rightarrow$ so not applicable to $c\bar{c}$ for example

These systems are:
$\rightarrow$ K^0-$\bar{K}^0$ ($d\bar{s}$), D^0-$\bar{D}^0$ ($c\bar{u}$), B^0-$\bar{B}^0$ ($d\bar{b}$), B_s^0-$\bar{B}_s^0$ ($s\bar{b}$)

they are subject to the mixing phenomenon via box diagrams:



Aside: Neutral Meson Systems

These systems are:

$\rightarrow$ K^0-$\overline{K}^0$ ($d\overline{s}$), D^0-$\overline{D}^0$ ($c\overline{u}$), B^0-$\overline{B}^0$ ($d\overline{b}$), B_s^0-$\overline{B}_s^0$ ($s\overline{b}$)

The neutral meson mixing corresponds to another case of misallignment between two sets of eigenstates:

Flavour eigenstates $\rightarrow$ defined flavour content:

$$M^0 \text{ and } \overline{M}^0$$

Mass eigenstates $\rightarrow$ defined masses $m_{1,2}$ and decay width $\Gamma_{1,2}$:

$$pM^0 \pm q\overline{M}^0$$

p & q complex coefficients that satisfy $|p|^2 + |q|^2 = 1$

In the famous case of kaons: $K_{S,L} \sim (1+\varepsilon)K^0 \pm (1-\varepsilon)\overline{K}^0$

In the formalism for the B mesons: $B_{L,H} \sim pB^0 \pm q\overline{B}^0$

Aside: Neutral Meson Systems

- Time-dependent Schrödinger eqn describes the evolution of the system:

$$i\frac{\partial}{\partial t}\begin{pmatrix} M^0 \\ \overline{M}^0 \end{pmatrix} = H\begin{pmatrix} M^0 \\ \overline{M}^0 \end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix} M^0 \\ \overline{M}^0 \end{pmatrix}$$

  - H is the hamiltonian; M and Γ are 2x2 hermitian matrices ($a_{ij} = \overline{a}_{ji}$)

$$M = ½ (H+H^\dagger) \text{ and } \Gamma = i(H-H^\dagger)$$

- CPT theorem imposes: $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$
  - particle and antiparticle have equal masses and lifetimes

Eigenvalues (μ) and mass (Δm) and lifetime ($\Delta\Gamma$) differences can be derived with this formalism:

$$\boldsymbol{\mu_{L,H} = m_{L,H} - i/2\,\Gamma_{L,H} = (M_{11} - i/2\,\Gamma_{11}) \pm (q/p)\,(M_{12} - i/2\,\Gamma_{12})}$$

$$\boldsymbol{\Delta m = m_H - m_L} \quad \text{and} \quad \boldsymbol{\Delta\Gamma = \Gamma_H - \Gamma_L}$$

$$\boldsymbol{(q/p)^2 = (M_{12}^* - i/2\,\Gamma_{12}^*)/(M_{12} - i/2\,\Gamma_{12})}$$

CPV Types for the B Meson System

- Define the quantity λ: $\lambda_{f_{CP}} = \frac{q}{p} \cdot \frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}}$

1. Indirect CP violation, or CPV in the mixing:

   $| q/p | \neq 1$

2. Direct CP violation, or CPV in the decays:

   $|\bar{A}/A | \neq 1$

   both neutral and charged B

3. CP violation in interference between mixing and decay: $\mathrm{Im}\lambda \neq 0$

   neutral B

Time evolution and CP violation

◎ Consider a B meson which is known to be a B (or $\overline{B}$) at t=0

◎ at t>0, the physical state has evolved in time with the amplitutes:

$$B^0_{phys}(t) = e^{-iMt} e^{-\Gamma t/2} \cos(\Delta m\, t/2)\, B^0 + i(q/p)\, e^{-iMt} e^{-\Gamma t/2} \sin(\Delta m\, t/2)\, \overline{B}^0$$

$$\overline{B}^0_{phys}(t) = i(q/p)\, e^{-iMt} e^{-\Gamma t/2} \sin(\Delta m\, t/2)\, B^0 + e^{-iMt} e^{-\Gamma t/2} \cos(\Delta m\, t/2)\, \overline{B}^0$$

mixing parameters

Δm = mass difference between the two mass eigenstates

→ in case of the B^0 mesons, difference between the heavy and light states

→ $\Delta m_d = 0.507 \pm 0.005\ \hbar/\mathrm{ps}$

Time evolution and CP violation

◎ If we consider that both B^0 and $\overline{B}^0$ can decay to the same final state and considering here a final state that is a CP eigenstate, then the time evolution of the physical system becomes:

$$f(B^0_{phys} \to f_{CP}, \Delta t) = \frac{\Gamma}{4} e^{-\Gamma|\Delta t|} \left[1 - S_{f_{CP}} \sin(\Delta m_d \Delta t) + C_{f_{CP}} \cos(\Delta m_d \Delta t)\right]$$

$$f(\bar{B}^0_{phys} \to f_{CP}, \Delta t) = \frac{\Gamma}{4} e^{-\Gamma|\Delta t|} \left[1 + S_{f_{CP}} \sin(\Delta m_d \Delta t) - C_{f_{CP}} \cos(\Delta m_d \Delta t)\right]$$

◉ direct CP violation $\quad C \neq 0 \quad$ $C_f (= -A_f) = \dfrac{1 - |\lambda_{f_{CP}}|^2}{1 + |\lambda_{f_{CP}}|^2}$

◉ CP violation in interference $\quad S \neq 0 \quad$ $S_f = \dfrac{2 Im \lambda_{f_{CP}}}{1 + |\lambda_{f_{CP}}|^2}$

Time-dependent CP asymmetries

- Ingredients of a time-dependent CP asymmetry measurement:
  - Isolate interesting signal B decay: B_{RECO}.
  - Identify the flavour of the non-signal B meson (B_{TAG}) at the time it decays.
  - Measure the spatial separation between the decay vertices of both B mesons: convert to a proper time difference $\Delta t = \Delta z / \beta\gamma c$;
  - fit for S and C.

- The time evolution of $B_{TAG} = \overline{B}^0 (B^0)$ is

$$f_{\pm}(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \left\{ 1 \pm \left[-\eta_f S \sin(\Delta m_d \Delta t) - C \cos(\Delta m_d \Delta t) \right] \right\}.$$

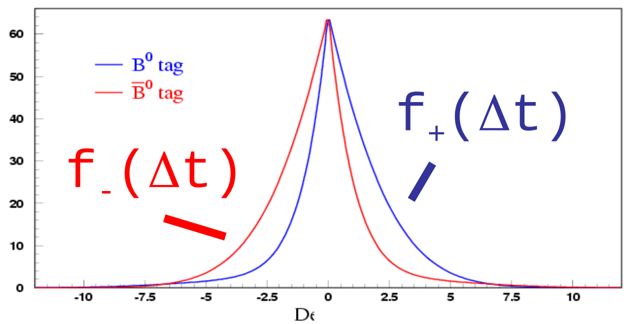
$\rightarrow$ B^0 lifetime $\tau_B = 1.530 \pm 0.009$ ps

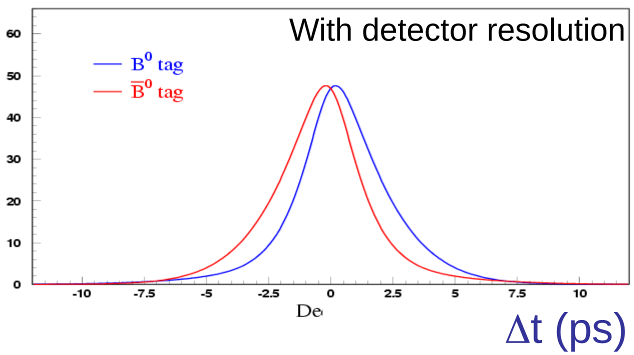
Time-dependent CP asymmetries

- Construct an asymmetry as a function of Δt:

$$\mathcal{A}(\Delta t) = \frac{\Gamma(\Delta t) - \overline{\Gamma}(\Delta t)}{\Gamma(\Delta t) + \overline{\Gamma}(\Delta t)}$$

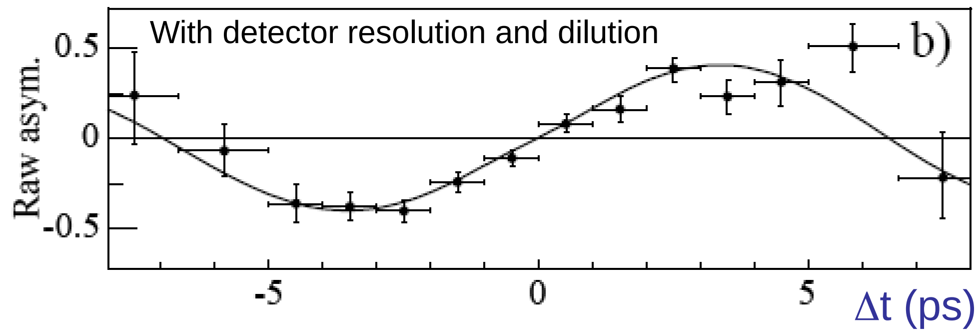
$$\mathcal{A}(\Delta t) = S\sin(\Delta m_d \Delta t) - C\cos(\Delta m_d \Delta t)$$





Experimental effects we need to include:

- Detector resolution on Δt.
- Dilution from flavour tagging



back-up