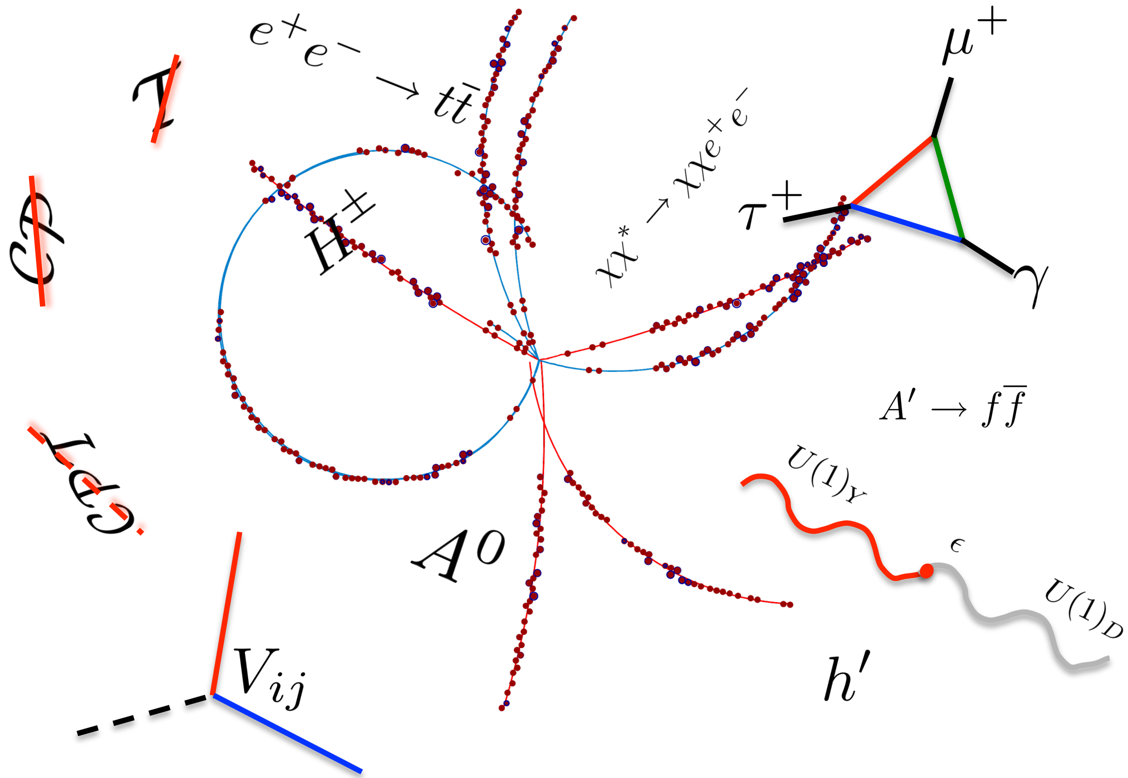
Flavour Physics at e^+e^- machines: Past/Present and Future (Part 3)



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IDPASC School, Valencia, May 2013

- There are three files:
  - 1) Introduction and formalism
  - 2) Results and future experiments
  - 3) Appendices (this one)

APPENDICES

APPENDIX I:

MORE ON α / Φ_2

APPENDIX II:

HOW DO I DO A GLOBAL FIT?
HOW DO I CONSTRAIN A NEW PHYSICS MODEL?

APPENDIX III:

CONVENTIONS

APPENDIX IV:

T VIOLATION IN B DECAY

Appendix I

- More on α / Φ_2
  - Some details regarding SU(3) and 3π determinations of α

Using SU(3)

(Appendix I)

- Can relate the penguin contribution in $K^{*+}\rho^0$ to that in $\rho^+\rho^-$:

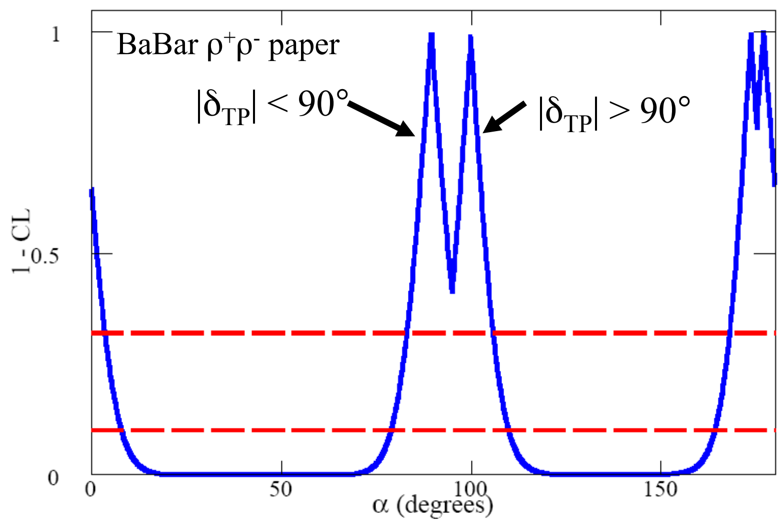
$$C_{\rm long} = \frac{2r\sin\delta_{\rm TP}\sin(\beta+\alpha)}{1-2r\cos\delta_{\rm TP}\cos(\beta+\alpha)+r^2},$$

$$S_{\rm long} = \frac{\sin 2\alpha + 2r\cos\delta_{\rm TP}\sin(\beta-\alpha) - r^2\sin 2\beta}{1-2r\cos\delta_{\rm TP}\cos(\beta+\alpha)+r^2}$$

$$\left(\frac{|V_{\rm cd}|f_\rho}{|V_{\rm cs}|f_{K^*}}\right)^2 \frac{\Gamma_{\rm L}(B^\pm \to K^{*0}\rho^+)}{\Gamma_{\rm L}(B^0 \to \rho^+\rho^-)} = \frac{Fr^2}{1-2r\cos\delta_{\rm TP}\cos(\beta+\alpha)+r^2}$$

The dominant $SU(3)$ breaking correction accounted for by F is the neglect of annihilation dia-

$$F = 0.9 \pm 0.6$$



- If we assume that $|\delta_{TP}| < 90°$:
- Relaxing this assumption:

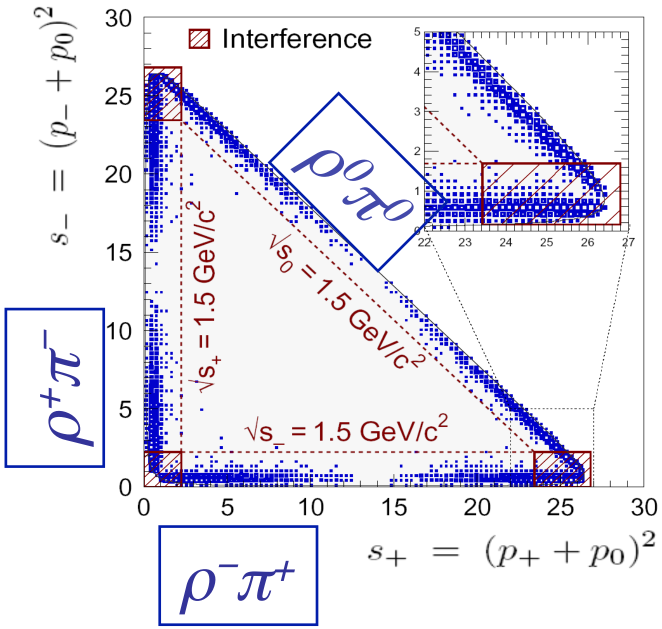
$$\alpha = [83.3, 105.8]^\circ$$

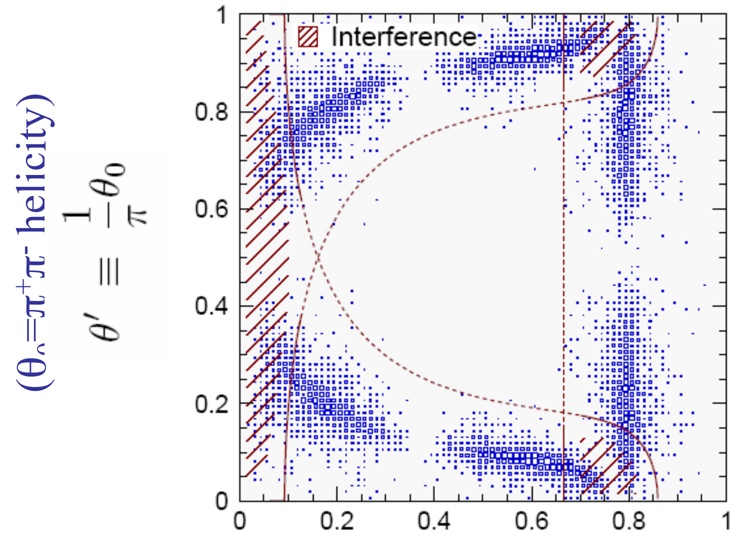
- Most precise determination of α!

M. Beneke, M. Gronau, J. Rohrer, and M. Spranger. *Phys. Lett. B* **638**, 68 (2006).

B→ρπ ($\pi^+\pi^-\pi^0$ Dalitz Plot) (Appendix I)

- Analyse a transformed Dalitz Plot to extract parameters related to α.
- Use the Snyder-Quinn method.





$$m' \equiv \frac{1}{\pi}\arccos\left(2\frac{m_0 - m_0^{\min}}{m_0^{\max} - m_0^{\min}} - 1\right)$$

($m_{\pi^+\pi^-}$)

$$|\mathcal{A}_{3\pi}^{\pm}(\Delta t)|^2 = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}}\left[|\mathcal{A}_{3\pi}|^2 + |\overline{\mathcal{A}}_{3\pi}|^2 \mp \left(|\mathcal{A}_{3\pi}|^2 - |\overline{\mathcal{A}}_{3\pi}|^2\right)\cos(\Delta m_d \Delta t) \pm 2\mathrm{Im}\left[\overline{\mathcal{A}}_{3\pi}\mathcal{A}_{3\pi}^*\right]\sin(\Delta m_d \Delta t)\right],$$

- Fit the time-dependence of the amplitudes in the Dalitz plot:

$B\to\rho\pi$ ($\pi^+\pi^-\pi^0$ Dalitz Plot) (Appendix I)

- The amplitudes are written in terms of Us and Is: 26 U and I parameters

$$|A_{3\pi}|^2 \pm |\overline{A}_{3\pi}|^2 = \sum_{\kappa\in\{+,0,-\}} |f_\kappa|^2 U_\kappa^\pm + 2\sum_{\sigma<\kappa\in\{+,0,-\}} \left(\mathrm{Re}\left[f_\kappa f_\sigma^*\right]U_\kappa^\pm - \mathrm{Im}\left[f_\kappa f_\sigma^*\right]U_{\kappa\sigma}^{\pm,\mathrm{Im}}\right)$$

$$\mathrm{Im}\left(\overline{A}_{3\pi}A_{3\pi}^*\right) = \sum_{\kappa\in\{+,0,-\}} |f_\kappa|^2 I_\kappa + \sum_{\sigma<\kappa\in\{+,0,-\}} \left(\mathrm{Re}\left[f_\kappa f_\sigma^*\right]I_{\kappa\sigma}^{\mathrm{Im}} + \mathrm{Im}\left[f_\kappa f_\sigma^*\right]I_{\kappa\sigma}^{\mathrm{Re}}\right)$$

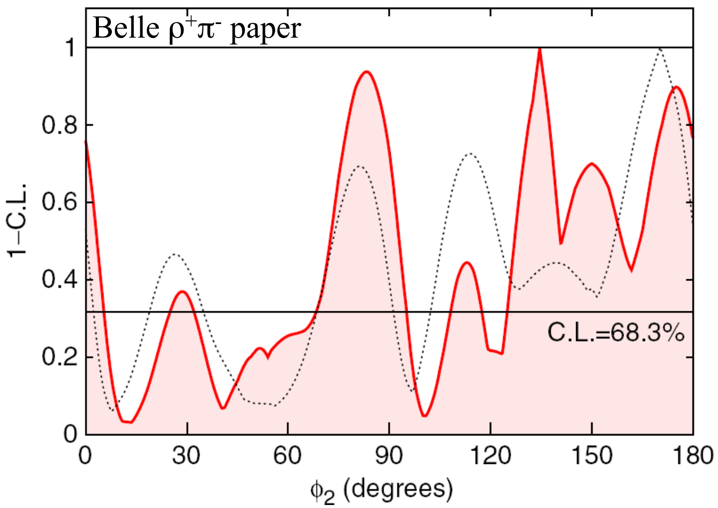
- Which are related to CP conserving and CP violating observables:

CP violating observables:

$$C = \frac{1}{2}\left(\frac{U_+^-}{U_+^+} + \frac{U_-^-}{U_-^+}\right) \qquad S = \frac{I_+}{U_+^+} + \frac{I_-}{U_-^+} \qquad A_{CP} = \frac{U_+^+ - U_-^+}{U_+^+ + U_-^+}$$

CP conserving observables:

$$\Delta C = \frac{1}{2}\left(\frac{U_+^-}{U_+^+} - \frac{U_-^-}{U_-^+}\right) \qquad \Delta S = \frac{I_+}{U_+^+} - \frac{I_-}{U_-^+}$$



Some features of this result:

- No region is excluded at 3σ significance.
- A high statistics measurement will help resolve ambiguities in the measured value of α.
- Results from the Dalitz analysis, and the pentagon analysis (solid) are more stringent than using the Dalitz analysis alone.

BaBar Result

(Appendix I)

- The determination of U's and I's is numerically robust, as is the determination of the Quasi-2-body approximation parameters.

- Given current data samples, conversion of these results into a 1-CL constraint on α is not robust. There is a finite probability of obtaining the best fit value corresponding to something other than the true value of the angle.

  - See arXiv:1304.3503 for details.

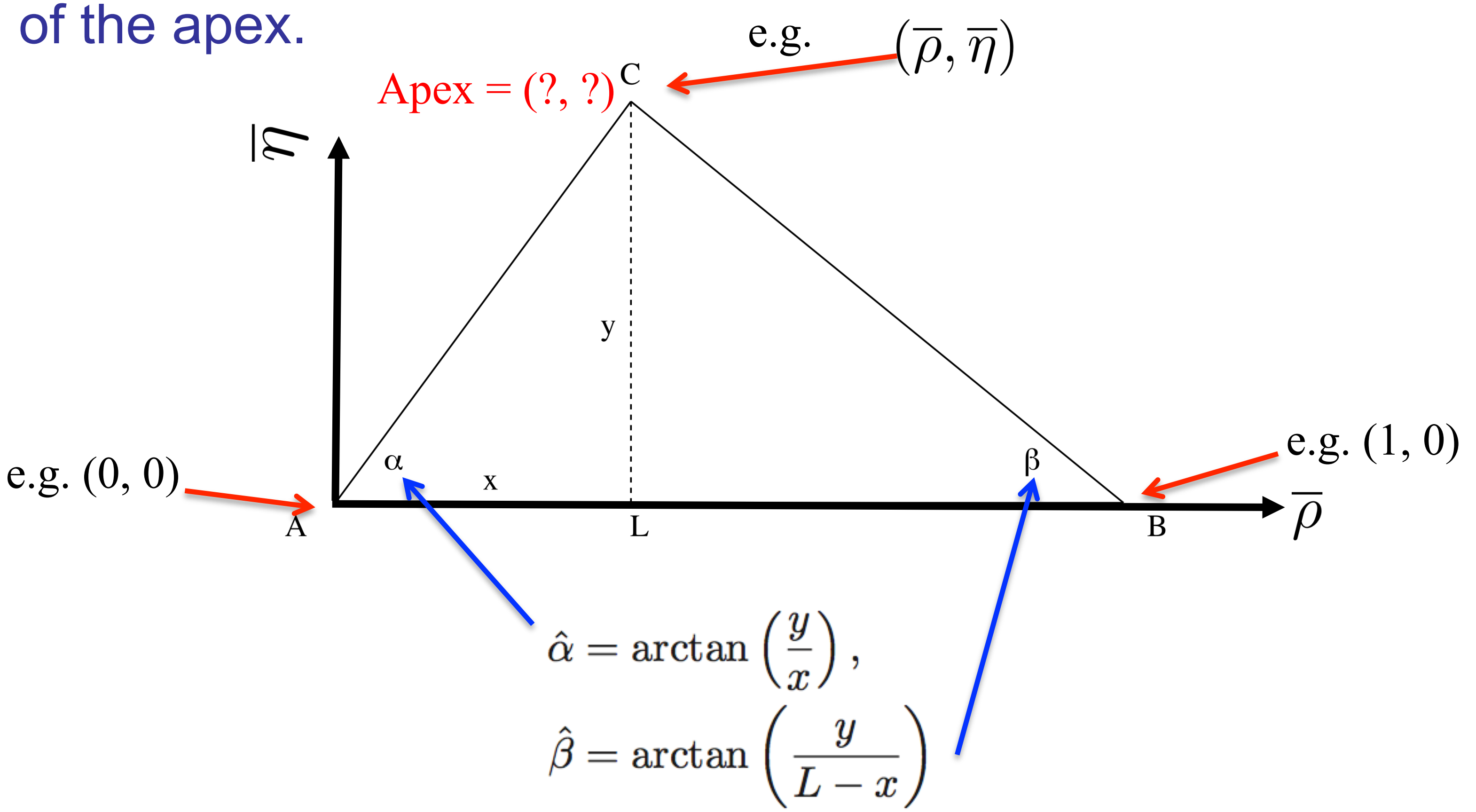
  - More data from Belle II is required to rectify this issue.

Appendix II

- How do I do a global fit?
  - There are three global fit groups:
    - CKM Fitter: http://ckmfitter.in2p3.fr/
    - UTfit: http://www.utfit.org/UTfit/
    - Unfit: arXiv:1301.5867 [Eigen et al.]
  - Two flavours of statistics:
    - Bayesian
      - Computational benefits, marginalize nuisance parameters, prior dependence etc.
    - Frequentist
      - "logical", but coverage needs to be understood, computationally expensive etc.
  - Many different inputs:
    - Theoretically straight forward (e.g. unitarity triangle angles)
    - Theoretically dependent (e.g. ε_K)
  - ... there is sufficient data to make meaningful global fits.

How do I do a global fit?

- The following illustrates how to locate the apex of a triangle with a known baseline. This is the equivalent of knowing two angles of the unitarity triangle, and determining an estimate of the apex.



These examples are derived from the book "Statistical Data Analysis for the physical sciences", AB, Cambridge University Press (2013).

How do I do a global fit? Frequentist (Appendix II)

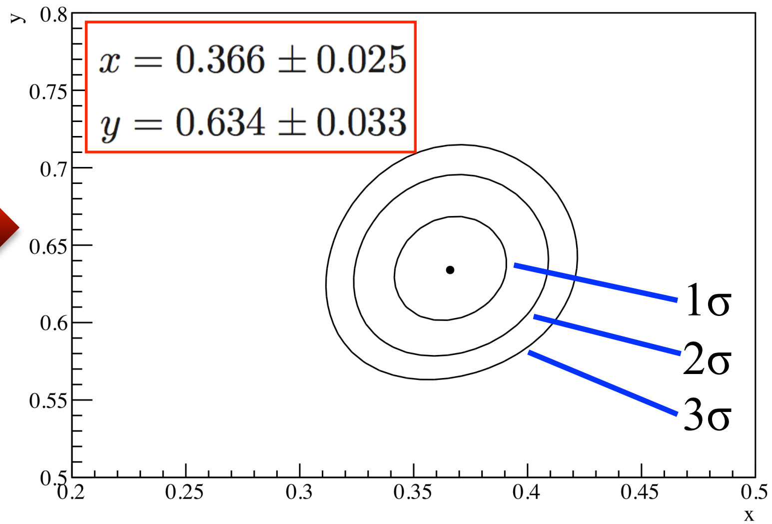
- Construct a X^2 from a number of constraints, and minimise this to obtain the most probable value, and an error ellipse (the confidence region) at some 1-CL value.

$$\chi^2 = \frac{(\alpha - \hat{\alpha})}{\sigma_\alpha^2} + \frac{(\beta - \hat{\beta})}{\sigma_\beta^2}$$

- For a given assumed *x* and *y* one can compute a value for the X^2, and then compute $P(X^2, \nu)$. From this one can obtain the desired result. e.g.

$$\begin{aligned} L &= 1 \\ \alpha &= (60 \pm 2)^\circ \\ \beta &= (45 \pm 2)^\circ \end{aligned}$$

$$\rho = \begin{pmatrix} 1.00 & -0.13 \\ -0.13 & 1.00 \end{pmatrix}$$



These examples are derived from the book "Statistical Data Analysis for the physical sciences", AB, Cambridge University Press (2013).

How do I do a global fit? Bayesian

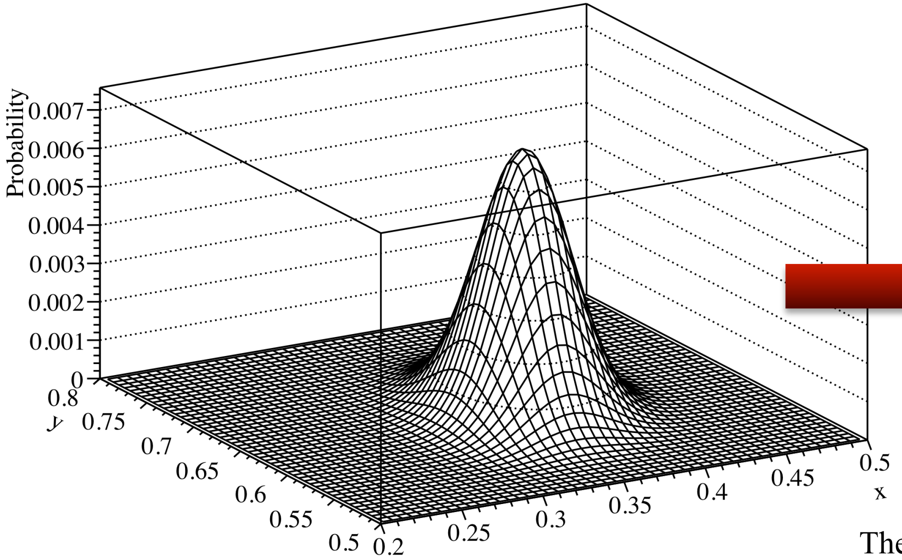
(Appendix II)

- Construct a priori probabilities (measurements = Gaussian?)
- Assume prior dependence
- Compute for a given (*x*, *y*)

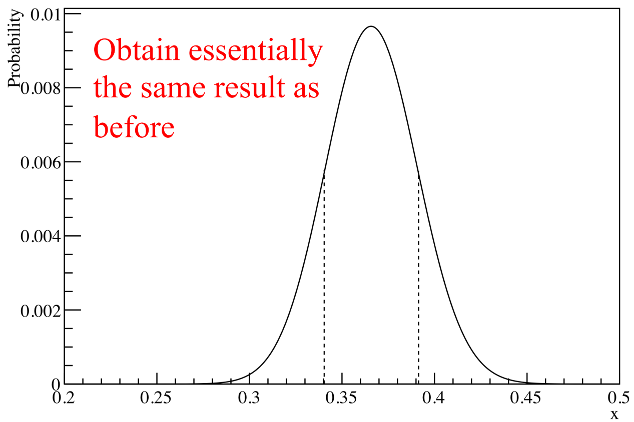
$$P(\hat{\alpha}|\alpha) = \frac{G(\hat{\alpha}, \alpha, \sigma_\alpha)P(\hat{\alpha})}{\int G(\hat{\alpha}, \alpha, \sigma_\alpha)d\underline{x}}$$

$$P(\hat{\beta}|\beta) = \frac{G(\hat{\beta}, \beta, \sigma_\beta)P(\hat{\beta})}{\int G(\hat{\beta}, \beta, \sigma_\beta)d\underline{x}}$$

$$\text{Probability} = P(\widehat{\alpha}|\alpha)P(\widehat{\beta}|\beta)$$



Integrate over variables/nuisance parameters to obtain marginal distribution for variable of interest



These examples are derived from the book "Statistical Data Analysis for the physical sciences", AB, Cambridge University Press (2013).

How do I do a global fit? Summary (Appendix II)

- Relatively straight forward to compute estimates for underlying parameters.
- Complications come into play when you have theoretical uncertainties, or a heavy theory input in converting an experimental observable into a theory parameter of interest.
- Both Frequentist and Bayesian approaches have issues that need to be addressed (in the case of Global CKM fits).
- With sufficient experimental data (i.e. precise enough measurements) the statistical approach taken should be (more or less) independent of the results obtained.
  - Differences in the way that theory uncertainties are treated may lead to differences in results.
  - Nuisance parameters and coverage may be issues for Frequentist treatment.
  - Prior dependence may be an issue for Bayesian treatment.

How do I constrain a new physics model?

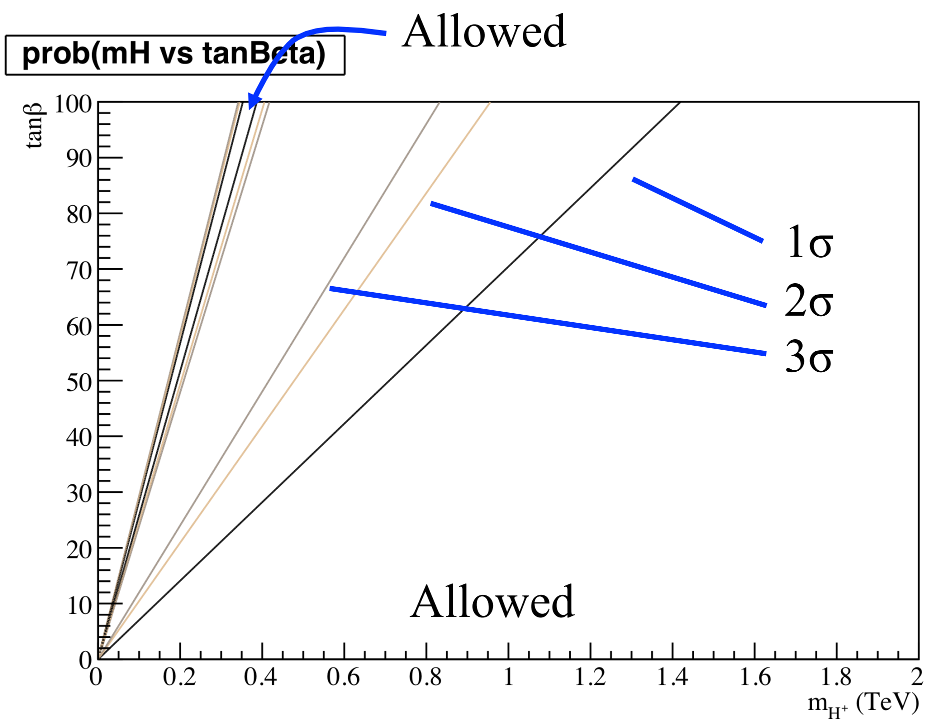
- These techniques can be applied to other scenarios in a straight forward way.
  - e.g. r_H for B decays into a lepton and neutrino final state, where the parameters fitted, or scanned through are tanβ and the charged Higgs mass in the case of a type-II 2HDM, and there is a single observable: r_H constraining these parameters.

ratio of Higgs vacuum expectation values

$$r_H = \left(1 - \frac{m_B^2}{m_{H^+}^2}\tan^2\beta\right)^2$$

Charged Higgs mass

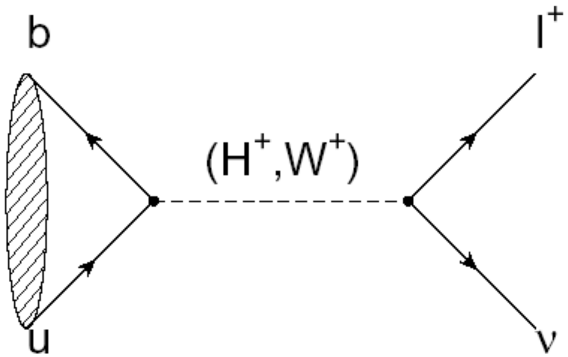
The ratio of branching ratios for SM+new physics, relative to the SM contribution.



The problem

- The decay $B^{\pm} \to \tau^{\pm}\nu$ has been measured, and can be compared with theoretical expectations.
- Measurement: $\mathcal{B}(B^{\pm} \to \tau^{\pm}\nu) = (1.15 \pm 0.23) \times 10^{-4}$
- Standard Model expectation:

$$\mathcal{B}(B^{\pm} \to \tau^{\pm}\nu)_{SM} = (1.01 \pm 0.29) \times 10^{-4}$$



$$r_H = \frac{\mathcal{B}_{SM+NP}}{\mathcal{B}_{SM}}$$

For a simple extension of the Standard Model, called the type II 2 Higgs Doublet Model we know that r_H depends on the mass of a charged Higgs and another parameter, β.

$$r_H = \left(1 - \frac{m_B^2}{m_H^2}\tan^2\beta\right)^2$$

What can we learn about m_H and tan β for this model?

- We can compute r_H from our knowledge of the measured and predicted branching fractions:

$$r_H = 1.14 \pm 0.40$$

- How can we use this to constrain m_H and tanβ?

$$r_H = \left(1 - \frac{m_B^2}{m_H^2} \tan^2\beta\right)^2$$

Method 1: χ^2 approach

(Appendix II)

- Construct a χ^2 in terms of r_H

Calculate using

$$r_H = \left(1 - \frac{m_B^2}{m_H^2}\tan^2\beta\right)^2$$

One has to select the parameter values.

From SM theory and experimental measurement

$$\chi^2 = \left(\frac{r_H - \widehat{r}_H(m_H, \tan\beta)}{\sigma_{r_H}}\right)^2$$

From SM theory and experimental measurement

Method 1: χ^2 approach

(Appendix II)

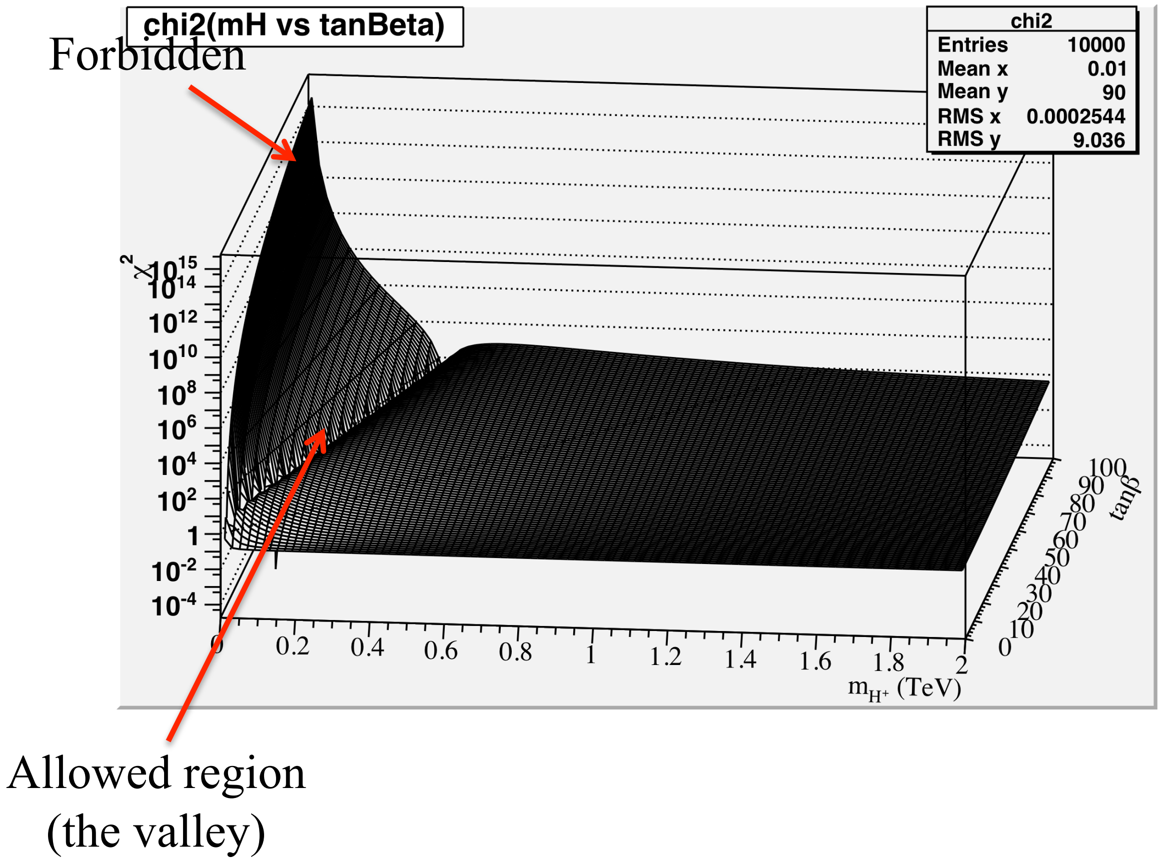
- For a given value of m_H and tanβ you can compute χ^2.

- e.g.

$$\begin{aligned} m_H &= 0.2\text{TeV} \\ \tan\beta &= 10 \\ \widehat{r}_H(m_H, \tan\beta) &= 0.93 \\ \chi^2 &= \left(\frac{1.14 - 0.93}{0.4}\right)^2 \\ &= 0.28 \end{aligned}$$

- So the task at hand is to scan through values of the parameters in order to study the behaviour of constraint on r_H.

Method 1: χ^2 approach



A largeχ^2 indicates a region of parameter space that is forbidden.

A small value is allowed.

In between we have to decide on a confidence level that we use as a cut-off.

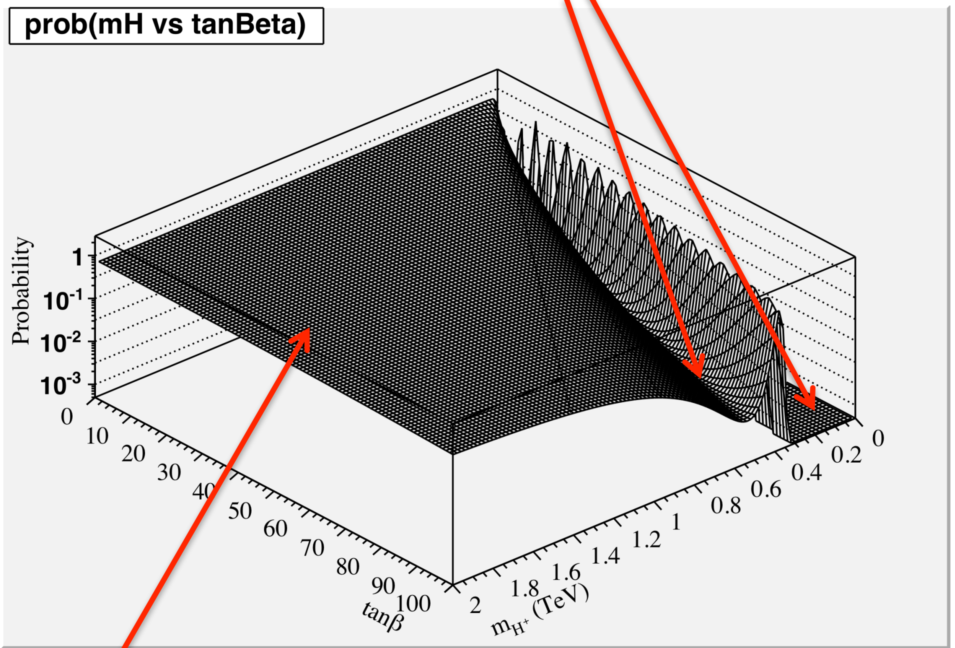
We really want to covert this distribution to a probability: so use the χ^2 probability distribution.

There are 2 parameters and one constraint (the data), so there are 2−1 degrees of freedom, i.e. ν=1

Method 1: χ^2 approach

(Appendix II)

Forbidden
(P ~ 0)



Allowed region
(P ~ 1)

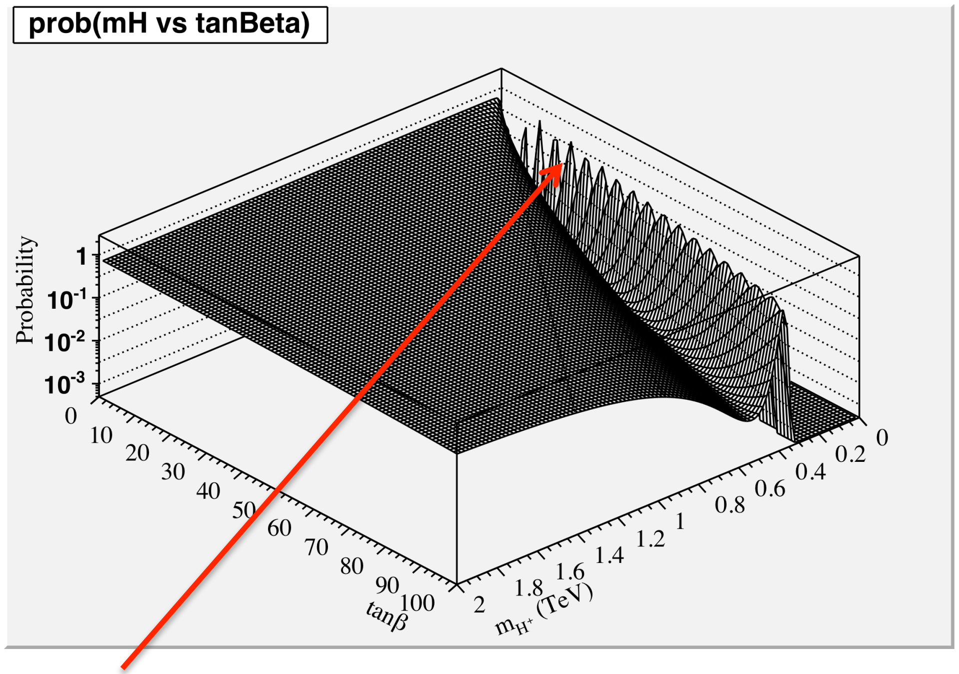
A value of P ~ 1 means that we have no constraint on the value of the parameters (i.e. they are allowed).

A small value of P, ~0 means that there is a very low (or zero) probability of the parameters being able to take those values (i.e. the parameters are forbidden in that region).

Typically one sets a 1−C.L. corresponding to 1 or 3 σ to talk about the uncertainty of a measurement, or indicate an exclusion region at that C.L.

Method 1: χ^2 approach

(Appendix II)



Artefact: a remnant of binning the data. For these plots there are 100 x 100 bins. As a result visual oddities can occur in regions where the probability (or χ^2) changes rapidly.
Solution: *finer binning!*

A value of P ~ 1 means that we have no constraint on the value of the parameters (i.e. they are allowed).

A small value of P, ~0 means that there is a very low (or zero) probability of the parameters being able to take those values (i.e. the parameters are forbidden in that region).

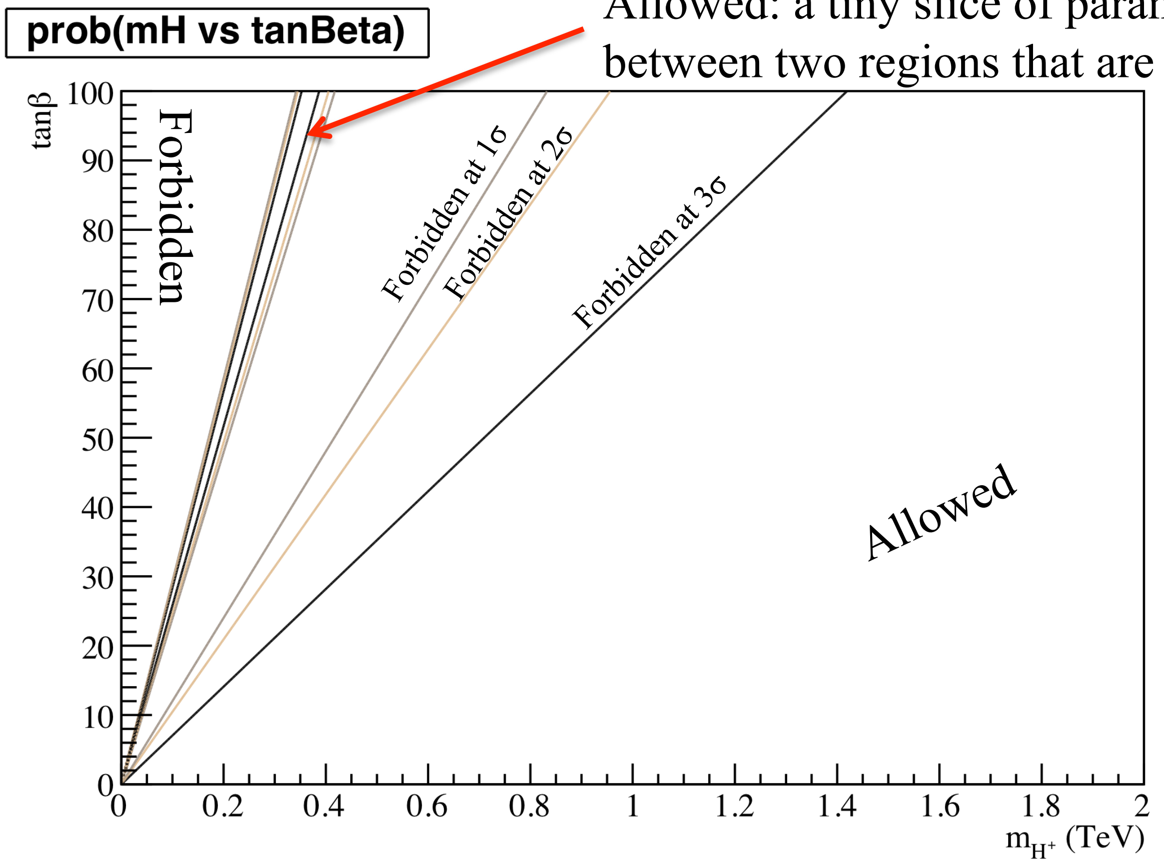
Typically one sets a 1−C.L. corresponding to 1 or 3 σ to talk about the uncertainty of a measurement, or indicate an exclusion region at that C.L.

Method 1: χ^2 approach

(Appendix II)

- A finer binning can be used to compute a 1-C.L. distribution. Here 1, 2 and 3σ intervals are shown.

Allowed: a tiny slice of parameter space is allowed between two regions that are forbidden



Appendix III

- Conventions
- This is a brief summary of conventions used on the different experiments for different (main) variable names.
- Unfortunately there is no uniformity to this process, and one has to get used to dual notations.

Description	*BABAR*	Belle
Unitarity Triangle angle	β	ϕ_1
Unitarity Triangle angle	α	ϕ_2
Unitarity Triangle angle	γ	ϕ_3
Beam constrained mass	m_{BC}	m_{ES}
Energy constrained mass	m_{ES}	m_{ES}
Energy difference	ΔE	ΔE
TDCPV sine coefficient	S	S
TDCPV cosine coefficient	C	$-A_{CP}$
Unassociated calorimeter energy	E_{EXTRA}	E_{ECL}

Appendix IV

- Testing T symmetry invariance
- This is a brief summary of results and ideas – see the lecture by J. Bernabeu for more details on the theoretical motivation.

http://www.economist.com/node/21561111
The Economist: 1st Sept 2012

The time-evolution of neutral meson systems is well understood, here one has to relate that information to T-conjugated pairs of decays in order to compute a T violating asymmetry.

A non-zero value of this asymmetry for any pair of T-conjugated decays would constitute T-symmetry non-invariance in that pairing.

Once can test the CKM matrix in a number of different ways using this approach.

Formalism

(Appendix IV)

- Need to test a T conjugate process, and compare a state $|i\rangle$ to some other state $|f\rangle$

$$A_T = \frac{P(|i\rangle \to |f\rangle) - P(|f\rangle \to |i\rangle)}{P(|i\rangle \to |f\rangle) + P(|f\rangle \to |i\rangle)}$$

c.f. CP asymmetries constructed from CP conjugate processes.

- The problem resides in identifying a T conjugate pair of processes that can be experimentally distinguished.
- ... and which could be used to experimentally test T symmetry non-invariance.
  - Given strong and EM conservation we want to identify weak decays that can be transformed under T to a conjugate state that can also be studied.

Time-evolution

(Appendix IV)

- Assuming $\Delta\Gamma=0$ (good for B_d decays)

$$C^{\pm}_{\alpha,\beta} = \frac{1-|\lambda|^2}{1+|\lambda|^2}$$

Superscripts:
+ = normal ordering
− = T reversed ordering

$$S^{\pm}_{\alpha,\beta} = \frac{2Im\lambda}{1+|\lambda|^2}$$

$$g^{\pm}_{\alpha,\beta}(\Delta t) \propto e^{-\Gamma\Delta t}\left[1 + C^{\pm}_{\alpha,\beta}\cos(\Delta m \Delta t) + S^{\pm}_{\alpha,\beta}\sin(\Delta m \Delta t)\right]$$

$$\alpha \in \{\ell^+, \ell^-\} \qquad \beta \in \{K_S, K_L\} \text{ i.e. } CP = \pm 1$$

- So one can relate the time-dependence to the weak structure of the decay (i.e. test the CKM formalism of the SM with an appropriate asymmetry observable).
- Need to account for mis-tag probability ω_α and detector resolution.

Event Selection: CP filters

(Appendix IV)

- The same as for the sin2β CPV measurement in *Phys.Rev. D**79**:072009 (2009)*

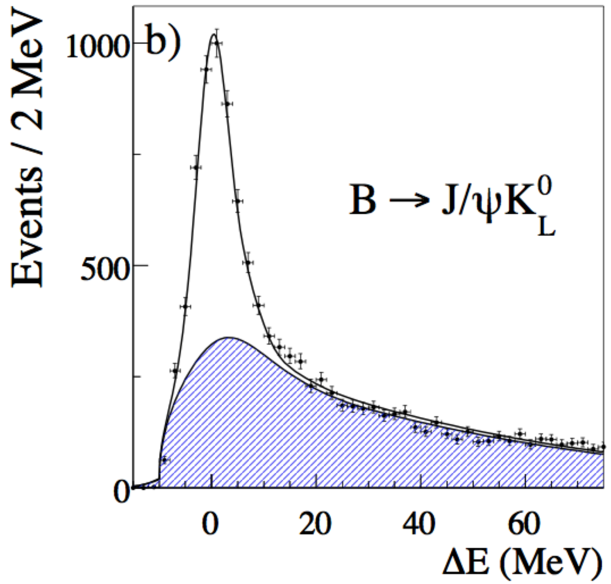
- CP even filter:

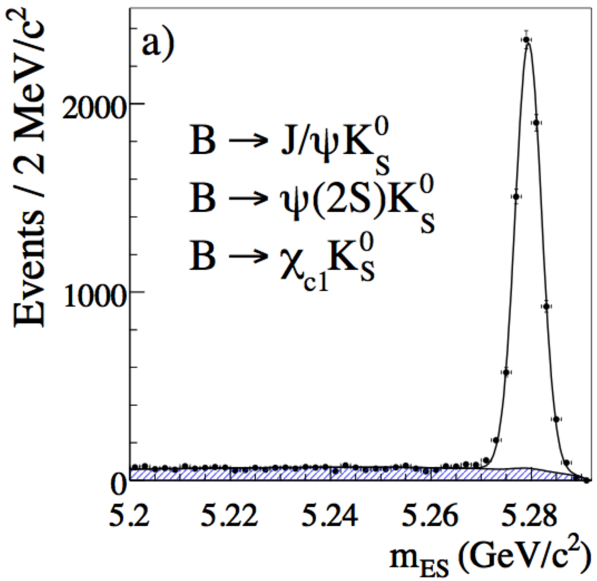
$$B \rightarrow J/\psi K_L$$

- CP odd filters:

$$\begin{aligned} B &\rightarrow J/\psi K_S \\ &\rightarrow \psi(2S) K_S \\ &\rightarrow \chi_{c1} K_S \end{aligned}$$

- Drop K* and η_c modes from the CP selection.





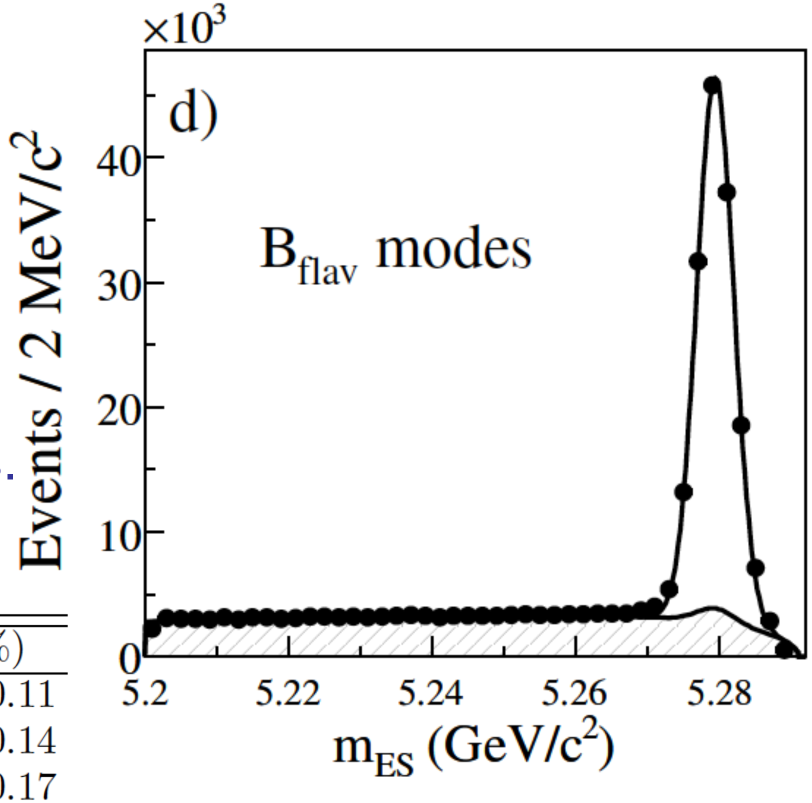
Event Selection: Flavor filters

- The same as for the sin2β CPV measurement in *Phys.Rev. D**79**:072009 (2009)*

- The set of "tag" modes used is:

$$B \rightarrow D^{(*)-}(\pi^{+},\ \rho^{+}, a_1^{+})$$

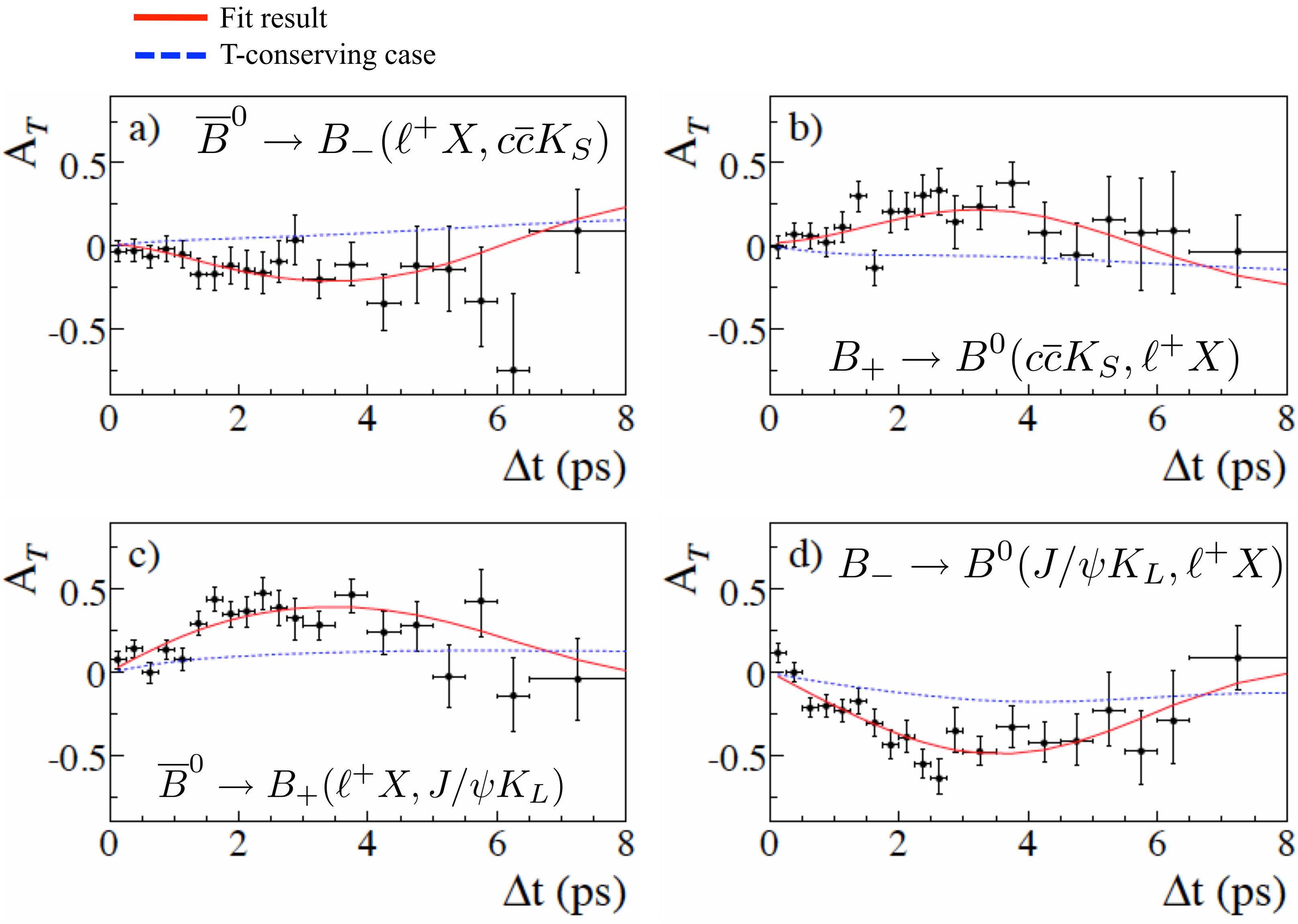
- which characterise "tag" performance and give the $B^0(\overline{B}^0)$ filter projections.



Category	ε (%)	w (%)	Δw (%)	Q (%)
Lepton	8.96 ± 0.07	2.8 ± 0.3	0.3 ± 0.5	7.98 ± 0.11
Kaon I	10.82 ± 0.07	5.3 ± 0.3	-0.1 ± 0.6	8.65 ± 0.14
Kaon II	17.19 ± 0.09	14.5 ± 0.3	0.4 ± 0.6	8.68 ± 0.17
KaonPion	13.67 ± 0.08	23.3 ± 0.4	-0.7 ± 0.7	3.91 ± 0.12
Pion	14.18 ± 0.08	32.5 ± 0.4	5.1 ± 0.7	1.73 ± 0.09
Other	9.54 ± 0.07	41.5 ± 0.5	3.8 ± 0.8	0.27 ± 0.04
All	74.37 ± 0.10			31.2 ± 0.3

Overall
$Q = 31.2\%$

Experimental results



Phys. Rev. Lett. 109, 211801 (2012) [arXiv:1207.5832]

Experimental results

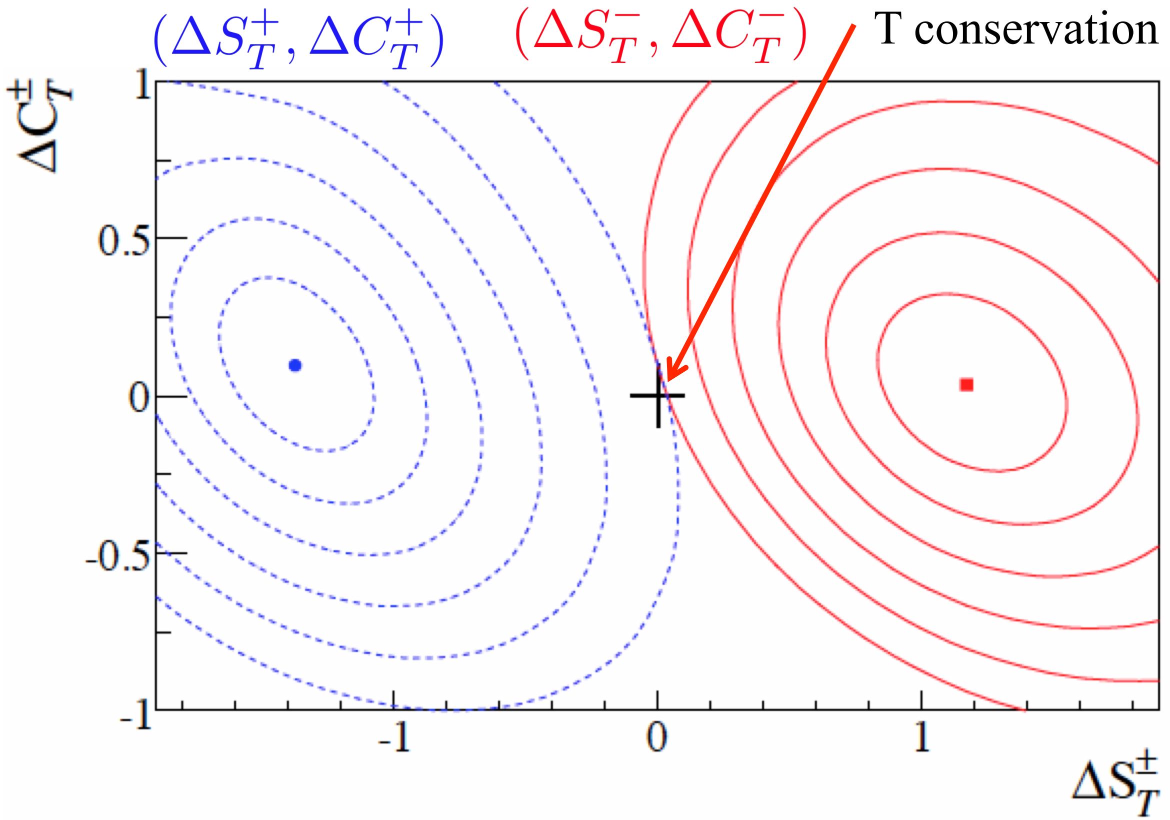
Parameter	Result
$\Delta S_T^+ = S^-_{\ell^-,K_L^0} - S^+_{\ell^+,K_S^0}$	$-1.37 \pm 0.14 \pm 0.06$
$\Delta S_T^- = S^+_{\ell^-,K_L^0} - S^-_{\ell^+,K_S^0}$	$1.17 \pm 0.18 \pm 0.11$
$\Delta C_T^+ = C^-_{\ell^-,K_L^0} - C^+_{\ell^+,K_S^0}$	$0.10 \pm 0.14 \pm 0.08$
$\Delta C_T^- = C^+_{\ell^-,K_L^0} - C^-_{\ell^+,K_S^0}$	$0.04 \pm 0.14 \pm 0.08$
$\Delta S_{CP}^+ = S^+_{\ell^-,K_S^0} - S^+_{\ell^+,K_S^0}$	$-1.30 \pm 0.11 \pm 0.07$
$\Delta S_{CP}^- = S^-_{\ell^-,K_S^0} - S^-_{\ell^+,K_S^0}$	$1.33 \pm 0.12 \pm 0.06$
$\Delta C_{CP}^+ = C^+_{\ell^-,K_S^0} - C^+_{\ell^+,K_S^0}$	$0.07 \pm 0.09 \pm 0.03$
$\Delta C_{CP}^- = C^-_{\ell^-,K_S^0} - C^-_{\ell^+,K_S^0}$	$0.08 \pm 0.10 \pm 0.04$
$\Delta S_{CPT}^+ = S^-_{\ell^+,K_L^0} - S^+_{\ell^+,K_S^0}$	$0.16 \pm 0.21 \pm 0.09$
$\Delta S_{CPT}^- = S^+_{\ell^+,K_L^0} - S^-_{\ell^+,K_S^0}$	$-0.03 \pm 0.13 \pm 0.06$
$\Delta C_{CPT}^+ = C^-_{\ell^+,K_L^0} - C^+_{\ell^+,K_S^0}$	$0.14 \pm 0.15 \pm 0.07$
$\Delta C_{CPT}^- = C^+_{\ell^+,K_L^0} - C^-_{\ell^+,K_S^0}$	$0.03 \pm 0.12 \pm 0.08$
$S^+_{\ell^+,K_S^0}$	$0.55 \pm 0.09 \pm 0.06$
$S^-_{\ell^+,K_S^0}$	$-0.66 \pm 0.06 \pm 0.04$
$C^+_{\ell^+,K_S^0}$	$0.01 \pm 0.07 \pm 0.05$
$C^-_{\ell^+,K_S^0}$	$-0.05 \pm 0.06 \pm 0.03$

- Observed level of T-violation balances CP violation.
- First direct measurement of T violation in B decays.
- Interpretation is unambiguous.

Phys. Rev. Lett. 109, 211801 (2012) [arXiv:1207.5832]

Experimental results

- Observation of T-violation can be seen in the following:



- Fit res g
  Gaussian errors).

CL = 0.317, 4.55×10^{-2}, 2.70×10^{-3}, 6.33×10^{-5}, 5.73×10^{-7}, 1.97×10^{-9}

$-2\Delta\ln\mathcal{L}$ = 2.3, 6.2, 11.8, 19.3, 28.7, 40.1

Phys. Rev. Lett. 109, 211801 (2012) [arXiv:1207.5832]

Experimental results

- Recall that $\Delta S^{\pm}$ are related to sin2β, so we can compare CP violation with T non-invariance for this parameter:

$$\Delta S^{-} \quad : \qquad \beta_{SM} = (17.9^{+3.9}_{-3.6})^{\circ}$$

$$\Delta S^{+} \quad : \qquad \beta_{SM} = (21.6^{+3.2}_{-2.9})^{\circ}$$

- c.f. beta measured from the standard CP analysis:

$$S \quad : \qquad \beta_{SM} = (21.7 \pm 1.2)^{\circ}$$

- As expected all results of β are in agreement with each other, however a more precise comparison of these results is called for.

This is my interpretation of the results.

- It was noted that one can remove the approximation that KL and KS are an orthonormal CP basis, by looking at B decays to two vector particle final states. AB, Inguglia, Zoccali, **arXiv:1302.4191**

Backup slides

Recoil reconstruction

- Technique adapted from CLEO (for D mesons) and applied to B mesons. Similar approach can be taken for top quarks.

Full Reconstruction

Signal decay to detected particles →

Hadronically reconstruct the tag final state of interest. Anything left must be from the other (signal) B in the final state.

Pro: Reconstruct whole event.

Con: Low efficiency (for B decays)

$$\epsilon \simeq 0.2 - 0.4\%$$

Signal decay includes neutrinos →

Hadronically reconstruct the tag final state of interest.
Anything left must be from the other (signal) B in the final state.

Pro: Reconstruct visible energy in whole event.

Con: Low efficiency (for B decays), and missing neutrino (but this can be used as a kinematic variable to suppress background).

Recoil reconstruction

- Technique adapted from CLEO (for D mesons) and applied to B mesons. Similar approach can be taken for top quarks.

Partial Reconstruction

$$\epsilon \simeq 0.7\%$$

Reconstruct semi-leptonic B decays as a B meson "tag".

Pro: High reconstruction efficiency, and missing mass can be used as a discriminating variable.

Con: Higher background than full reconstruction approach, and event can't be fully reconstructed.

Recoil reconstruction

- Technique adapted from CLEO (for D mesons) and applied to B mesons. Similar approach can be taken for top quarks.

Partial Reconstruction

$$\epsilon \simeq 0.7\%$$

Reconstruct semi-leptonic B decays as a B meson "tag".

Pro: High reconstruction efficiency, and missing mass can be used as a discriminating variable.

Con: Higher background than full reconstruction approach, and event can't be fully reconstructed.

The efficiencies noted are typical values used in papers during the life of the B Factories. Recently more complicated hadronic and semi-leptonic tag algorithms have been used, with higher efficiencies.