Scientific Measurement

PHY-103

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Lecture 8 - Weighted Means & Systematic Uncertainties

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- Fitting n data points to a theory with m parameters - calculate:

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{y_i - f(x_i)}{\sigma_i} \right)^2$$

expect $\frac{\chi^2}{ndf} \approx 1$ $\quad ndf$ = No. degrees of freedom = $n - m$

If χ^2/ndf is much larger than 1:

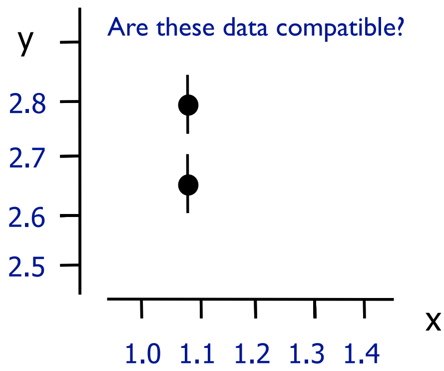
uncertainties on data points underestimated??

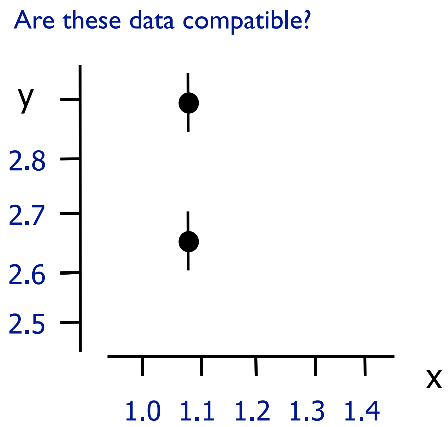
theory incompatible with data?

If χ^2/ndf is much smaller than 1:

uncertainties on data points possibly overestimated??

- Two independent measurements of the same quantity
  How can we decide if measurements are compatible?





Calculate the number of Std. Deviations difference between measurements d_1 & d_2

This is like calculating the uncertainty on (d_1-d_2) in units of total error on (d_1-d_2)

$$D = |d_1 - d_2|$$

$$\sigma_D^2 = \sigma_{d_1}^2 + \sigma_{d_2}^2 \quad \text{from propagation of errors formula}$$

$$N_{Std.Dev} = \frac{D}{\sigma_D} = \frac{|d_1 - d_2|}{\sqrt{\sigma_{d_1}^2 + \sigma_{d_2}^2}}$$

So D=0 would indicate compatible data
But measurements with large errors can also be compatible!

<u>Rule of Thumb</u>
$N_{Std.Dev}$ < 3 implies data are compatible (with probability of 99.7%*)
$N_{Std.Dev}$ > 3 implies data are not compatible

Note: statements of compatibility depend on uncertainties of the measurements!

Note: this is equivalent to calculating the χ^2 between data points

*Assuming independent Gaussian uncertainties

If data are compatible (and only if they are compatible!) can we combine measurements

Average several measurements with different uncertainties to obtain a more precise average

Construct an average in which more precise data count for more - i.e. have larger weight

Define weights for each measurement w_i

$$w_i = \frac{1}{\sigma_i^2}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} = \frac{\sum_{i=1}^{n} \frac{x_i}{\sigma_i^2}}{\sum_{i=1}^{n} \frac{1}{\sigma_i^2}}$$

error on weighted mean is $\sigma_{\bar{x}} = \sqrt{\frac{1}{\sum_{i=1}^{n} w_i}}$

Data with large error has small weight
Data with small error has large weight

In first lecture I stated uncertainties are of two types:

Statistical & Systematic

Total measurement uncertainty is a combination of the two

We have looked at statistical uncertainty

Have examined how to combine uncertainties through error propagation

What are systematic uncertainties?

Arise from <u>assumptions</u> you have made & possible errors due to your experimental <u>method</u>

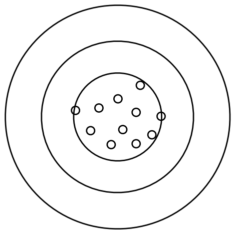
Much more detailed knowledge of experiment required to understand systematic effects

Judging systematic errors can be something of an “art”

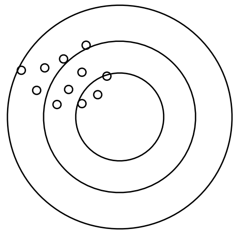
Often can exclude treatment of systematics if statistical error is very large

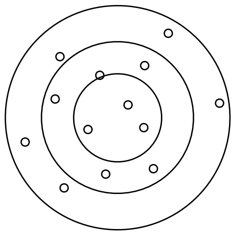
Example of random (statistical) errors and systematic errors in target practice

small random error
small systematic error

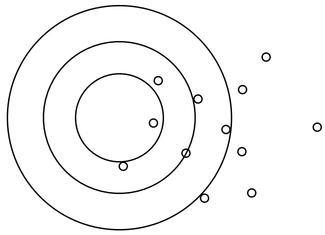


small random error
large systematic error





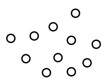
large random error
small systematic error



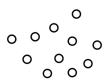
large random error
large systematic error

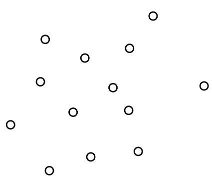
In practice we do not know the true value of the quantity we want to measure!

small random error
?? systematic error

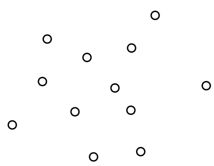


small random error
?? systematic error





large random error
?? systematic error



large random error
?? systematic error

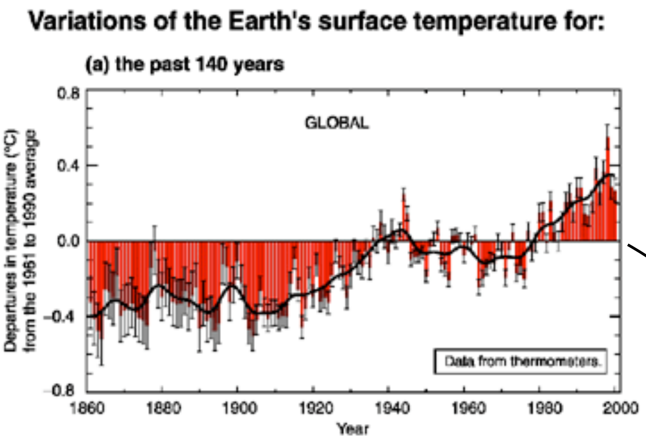
Examples of systematic uncertainties in experiments 1-4

Experiment 1: Measurement of g, Acceleration due to gravity
calibration of ruler (±~1mm)
calibration of stopwatch (±~0.2 s)
your own reaction time (±~0.1 s)

Experiment 2: Power output of electrical circuit
precision of resistors used in circuit (±1-5%)
precision of voltmeter calibration (±1 % ??)

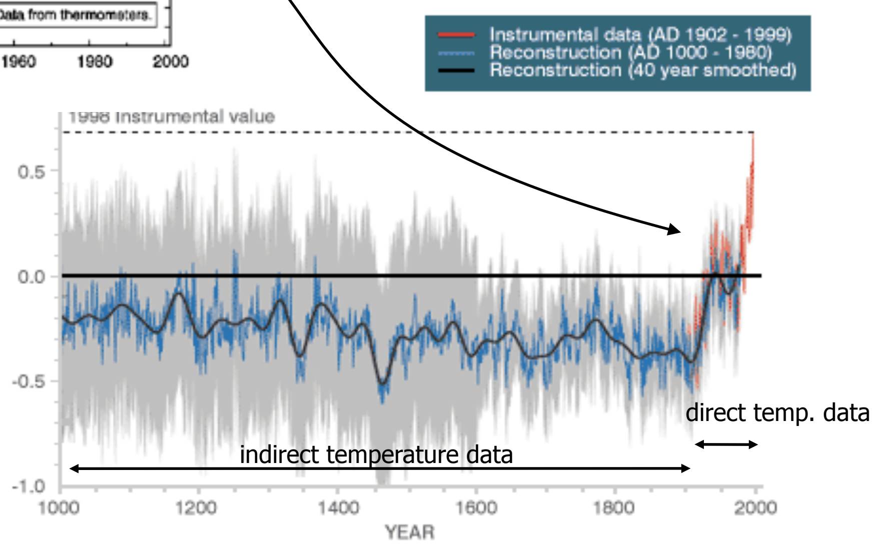
Experiment 3: Extension of rubber band
precision of masses used (± ~5%)
precision of vernier calibration (±~0.1mm)

Experiment 4: Radioactivity
efficiency of Geiger counter (±~1% ??)
precision of ruler calibration (±~1mm)



Earth's temperature deviations over last 150 years from thermometer data

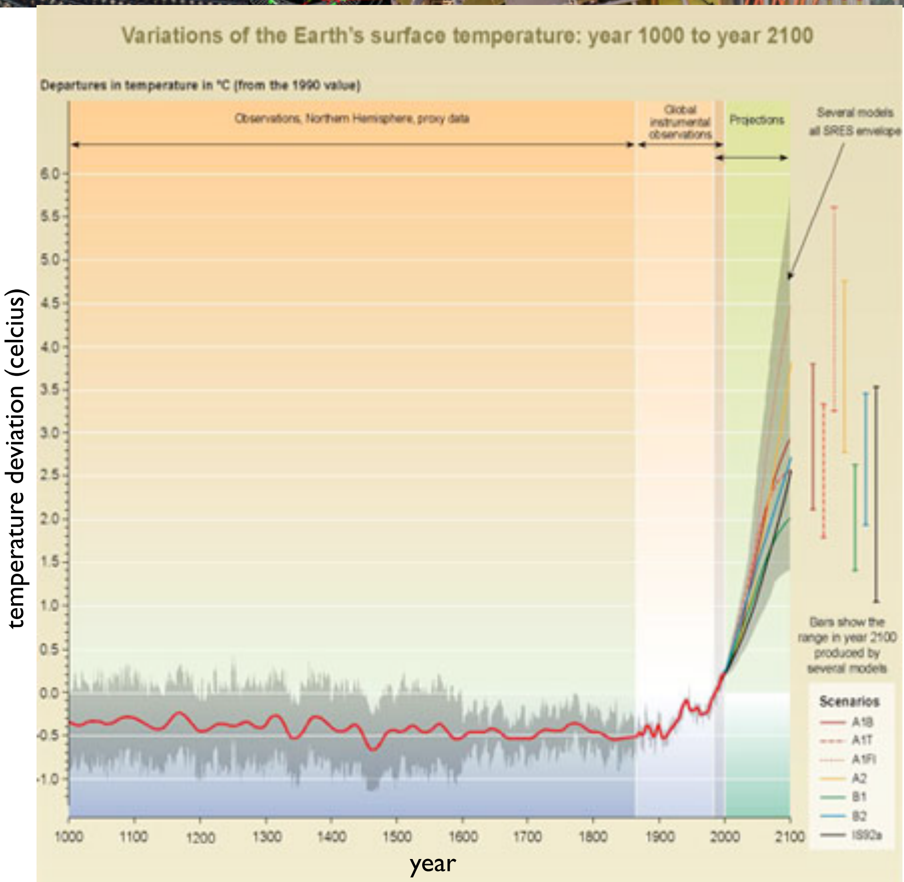
Each data point is a weighted mean of many thermometer readings



Earth's temperature deviations over last 1000 years from tree rings and ice core data.

Grey region represents uncertainty

Cross check direct & indirect data - "calibration check"



temperature predictions including uncertainties on model extrapolation till year 2100

Two lab reports to be written up:

experiment 5

experiment 8-12 deadline January

Aim:

teach professional standards

communicate information

be precise

be concise - I DO NOT want 30 page reports!!!

Another experimentalist should be able to reproduce the results from reading your report

Structure:

Title

Authors / Affiliation

Abstract

Theory / Principles of operation

Experimental Method

Results including tables & graphs & error analysis

Conclusion

References

You will be marked for Presentation:

- No excuse for poor presentation
- Use word processor
- No spelling mistakes - use spell checker
- Check your grammar: four/for, their/there, to/too etc...
- If english is not your first language - find a friend to check it
- Style - NOT CHATTY - dispassionate and professional

- Use equation editor
- Use the symbol font for + - x ± ¸ » p q etc...
- Variable names should be italicised

- Ensure you have page numbers
- Check superscripts and subscripts e.g. 10^2

- Make sure you quote an appropriate number of sig.figs

- Don't forget units!

Usually papers are written in impersonal voice:
"The experiment was set-up..."
"Measurements were taken.."

Not:
"I set-up the experiment..."
"We took measurements..."

Abstract:

- 1 short paragraph ~100 words
- explain what you did
- give main result and conclusion

Theory:

- Overview of background - but don't repeat, use references
- Give any derivations if needed

Method:

- Be PRECISE - define all your quantities
- Think of the logical flow of method, not the chronological order!

Results:

- Show the raw data
- Estimate uncertainties
- Graphs should have axis labels, legend, units, caption, title
- Do not show pages and pages of calculations here - reference

Conclusion:

What did you learn?

References:

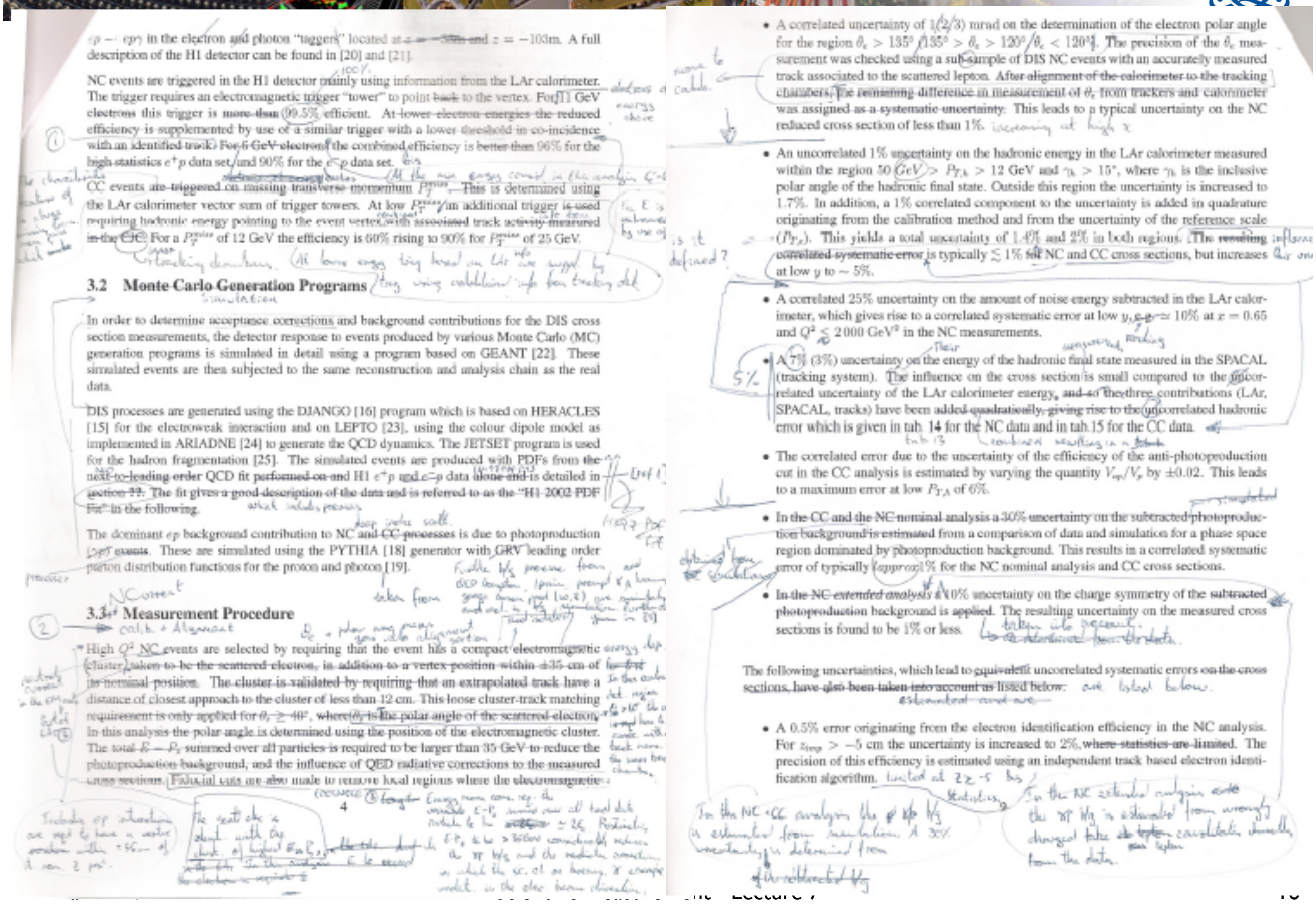
List all refs at the end

Collect plastic binder from Pete / Saqib

Do not trust your printer!!!
It will break 30 mins before the hand in deadline
I guarantee this!
Ensure you print it out LONG before the deadline
Test the print out to ensure fonts are correctly reproduced on paper

I also guarantee that several of you will accidentally delete the document
Keep a backup on a USB stick

No deadline extensions will be given



$ep \rightarrow ep\gamma$ in the electron and photon "taggers" located at $z = -33$m and $z = -103$m. A full description of the H1 detector can be found in [20] and [21].

NC events are triggered in the H1 detector mainly using information from the LAr calorimeter. The trigger requires an electromagnetic trigger "tower" to point back to the vertex. For 11 GeV electrons this trigger is more than 99.5% efficient. At lower electron energies the reduced efficiency is supplemented by use of a similar trigger with a lower threshold in co-incidence with an identified track. For 6 GeV electrons the combined efficiency is better than 96% for the high statistics e^+p data set and 90% for the e^-p data set.

CC events are triggered on missing transverse momentum P_T^{miss}. This is determined using the LAr calorimeter vector sum of trigger towers. At low P_T^{miss} an additional trigger is used requiring hadronic energy pointing to the event vertex with associated track activity measured in the CJC. For a P_T^{miss} of 12 GeV the efficiency is 60% rising to 90% for P_T^{miss} of 25 GeV.

3.2 Monte Carlo Generation Programs

In order to determine acceptance corrections and background contributions for the DIS cross section measurements, the detector response to events produced by various Monte Carlo (MC) generation programs is simulated in detail using a program based on GEANT [22]. These simulated events are then subjected to the same reconstruction and analysis chain as the real data.

DIS processes are generated using the DJANGO [16] program which is based on HERACLES [15] for the electroweak interaction and on LEPTO [23], using the colour dipole model as implemented in ARIADNE [24] to generate the QCD dynamics. The JETSET program is used for the hadron fragmentation [25]. The simulated events are produced with PDFs from the next-to-leading order QCD fit performed on and H1 e^+p and e^-p data alone and is detailed in section ??. The fit gives a good description of the data and is referred to as the "H1 2002 PDF Fit" in the following.

The dominant ep background contribution to NC and CC processes is due to photoproduction (γp) events. These are simulated using the PYTHIA [18] generator with GRV leading order parton distribution functions for the proton and photon [19].

3.3 Measurement Procedure

High Q^2 NC events are selected by requiring that the event has a compact electromagnetic cluster, taken to be the scattered electron, in addition to a vertex position within ±35 cm of its nominal position. The cluster is validated by requiring that an extrapolated track have a distance of closest approach to the cluster of less than 12 cm. This loose cluster-track matching requirement is only applied for $\theta_e \geq 40°$, where θ_e is the polar angle of the scattered electron. In this analysis the polar angle is determined using the position of the electromagnetic cluster. The total $E - P_z$ summed over all particles is required to be larger than 35 GeV to reduce the photoproduction background, and the influence of QED radiative corrections to the measured cross sections. Fiducial cuts are also made to remove local regions where the electromagnetic

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- A correlated uncertainty of 1(2/3) mrad on the determination of the electron polar angle for the region $\theta_e > 135°$ / $135° > \theta_e > 120°$ / $\theta_e < 120°$. The precision of the θ_e measurement was checked using a sub-sample of DIS NC events with an accurately measured track associated to the scattered lepton. After alignment of the calorimeter to the tracking chambers, the remaining difference in measurement of θ_e from trackers and calorimeter was assigned as a systematic uncertainty. This leads to a typical uncertainty on the NC reduced cross section of less than 1%.
- An uncorrelated 1% uncertainty on the hadronic energy in the LAr calorimeter measured within the region 50 GeV $> P_{T,h} >$ 12 GeV and $\gamma_h > 15°$, where γ_h is the inclusive polar angle of the hadronic final state. Outside this region the uncertainty is increased to 1.7%. In addition, a 1% correlated component to the uncertainty is added in quadrature originating from the calibration method and from the uncertainty of the reference scale ($P_{T,e}$). This yields a total uncertainty of 1.4% and 2% in both regions. The resulting correlated systematic error is typically $\lesssim 1$% for NC and CC cross sections, but increases at low y to ~ 5%.
- A correlated 25% uncertainty on the amount of noise energy subtracted in the LAr calorimeter, which gives rise to a correlated systematic error at low y, e.g. ≃ 10% at $x = 0.65$ and $Q^2 \lesssim 2\,000$ GeV^2 in the NC measurements.
- A 7% (3%) uncertainty on the energy of the hadronic final state measured in the SPACAL (tracking system). The influence on the cross section is small compared to the uncorrelated uncertainty of the LAr calorimeter energy, and so the three contributions (LAr, SPACAL, tracks) have been added quadratically, giving rise to the uncorrelated hadronic error which is given in tab. 14 for the NC data and in tab.15 for the CC data.
- The correlated error due to the uncertainty of the efficiency of the anti-photoproduction cut in the CC analysis is estimated by varying the quantity V_{ap}/V_p by ±0.02. This leads to a maximum error at low $P_{T,h}$ of 6%.
- In the CC and the NC nominal analysis a 30% uncertainty on the subtracted photoproduction background is estimated from a comparison of data and simulation for a phase space region dominated by photoproduction background. This results in a correlated systematic error of typically (approx.)1% for the NC nominal analysis and CC cross sections.
- In the NC extended analysis a 10% uncertainty on the charge symmetry of the subtracted photoproduction background is applied. The resulting uncertainty on the measured cross sections is found to be 1% or less.

The following uncertainties, which lead to equivalent uncorrelated systematic errors on the cross sections, have also been taken into account as listed below.

- A 0.5% error originating from the electron identification efficiency in the NC analysis. For $z_{imp} > -5$ cm the uncertainty is increased to 2%, where statistics are limited. The precision of this efficiency is estimated using an independent track based electron identification algorithm.