Scientific Measurement

PHY-103

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Lecture 7 - Hypothesis Testing

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How well do we know our errors?

Why do we not quote results as 8.214 ± 0.132 ?

We know σ - what is the error on this quantity?

- the error of the error

$$\frac{\sigma_\sigma}{\sigma} = \frac{1}{\sqrt{2n}}$$

Depends only on number of measurements *n*

For n=10 $\frac{\sigma_\sigma}{\sigma} \approx 0.22 = 22\%$

For n=50 $\frac{\sigma_\sigma}{\sigma} \approx 0.1 = 10\%$

It is difficult to know error better than 10% !

Never quote error to more than 2 sig.figs!

Rule of Thumb:

Look at first 3 sig.figs of the error

quote 2 sig.figs if < 355

quote 1 sig.fig if > 355

Examples:

0.713 ± 0.059 should be given as 0.71 ± 0.06

12.1749 ± 0.0112 should be given as 12.175 ± 0.011

0.07341 ± 0.0462 should be given as 0.73 ± 0.05

Keep the precision the same for error and central value

i.e. same number of decimal places

e.g. never 123.57 ± 2 !!!

Fitting Data

Scientists confront data with theory

If data and theory disagree? Theory is wrong!

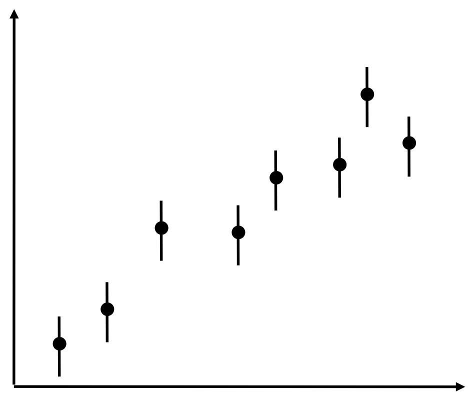
If data and theory agree? Theory is right

No!

Scientific data can only falsify theories.

Theory may be correct, but parameters of theory may be wrong

Strongest statement you can make is that data and theory are consistent



Theory predicts y=f(x)

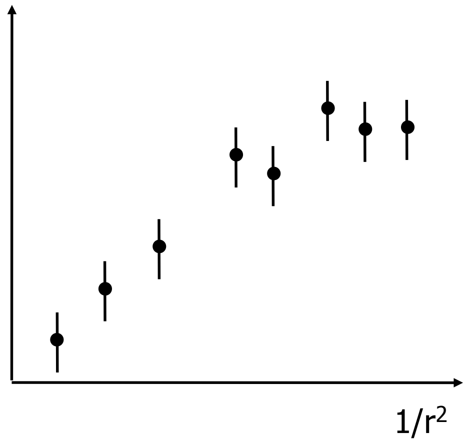
Theory may have a free parameter

$$F = G\frac{m_1 m_2}{r^2}$$

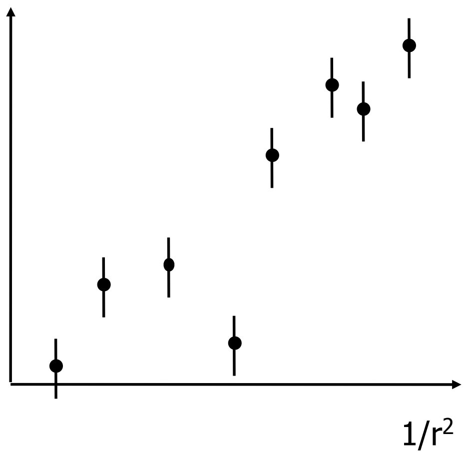
G is not predicted!

Slope of plot F vs $1/r^2$ determines G

What if we measure data like the following?



$$F = G\frac{m_1 m_2}{r^2}$$



We fit the data to the theory and determine the best value for G
i.e. we extract the best value for a and b in a straight line fit y=ax+b

Consider n data points (x_i,y_i) with error σ_i

Assume error on x $\sigma_x=0$

We calculate residuals of data to theory:

$$E = \sum_{i=1}^{n} [y_i - f(x_i)]^2$$

$$= \sum_{i=1}^{n} [y_i - (ax_i + b)]^2$$

Letting $\frac{\partial E}{\partial a} = 0$ and $\frac{\partial E}{\partial b} = 0$

after some algebra....

$$a = \frac{n\sum_i x_i y_i - \sum_i x_i \sum_i y_i}{n\sum_i x_i^2 - \left(\sum_i x_i\right)^2}$$

$$b = \frac{n\sum_i y_i - a\sum_i x_i}{n}$$

no need to remember
computers do it for you!

x_i	y_i (±1.5)	r_i (residual)	r_i/σ_i
0	2.6	-2.11	-1.41
1	8.1	+1.06	+0.71
2	9.1	-0.27	-0.18
3	12.5	+0.80	+0.53
4	14.7	+0.67	+0.45
5	18.9	+2.54	+1.69
6	17.6	-1.09	-0.73
7	18.9	-2.12	-1.41
8	24.5	+1.15	+0.77
9	24.9	-0.28	-0.19

Errors are ±1.5 on the y_i measurements

calculating: a=2.327
b=4.708

thus y = 4.71 + 2.33 x

residuals: $r_i = y_i - f(x_i)$

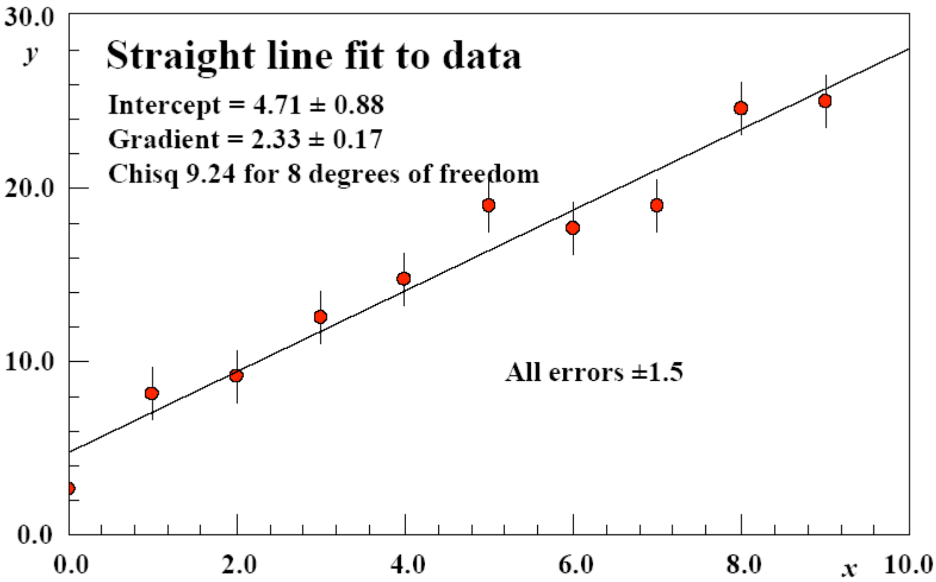
This is the deviation of any one measurement from the calculated slope

Often the quality of the fit is stated in terms of the χ^2 test

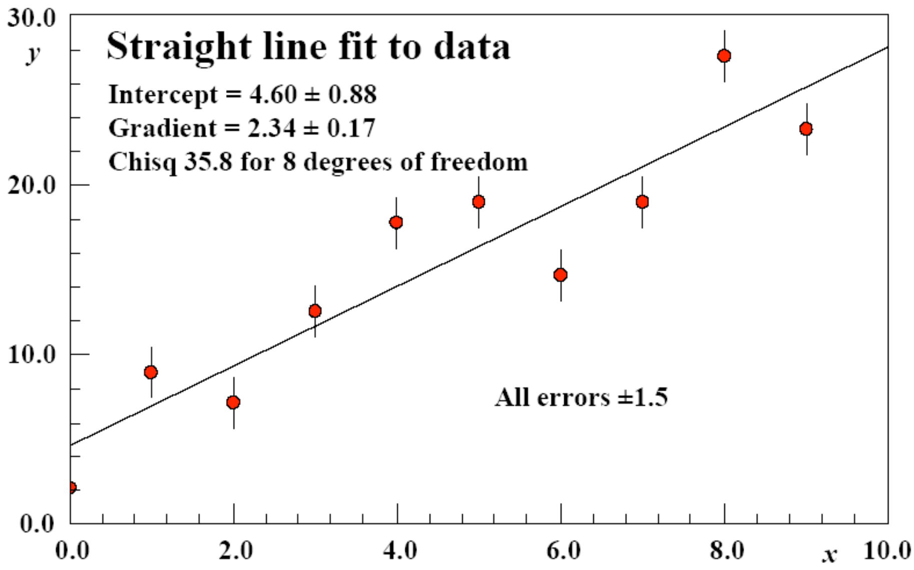
Think of it as summed residual in units of the error

$$\boxed{\chi^2 = \sum_{i=1}^{n} \left(\frac{r_i}{\sigma_i} \right)^2 = \sum_{i=1}^{n} \left(\frac{y_i - f(x_i)}{\sigma_i} \right)^2}$$

For a good fit, expect average deviation to be about 1σ, i.e. $\chi^2 \approx n$ i.e. $\chi^2/n \approx 1$



This is the data plotted with PhysPlot
Could fit using a polynomial e.g. quadratic with 3 parameters, or cubic with 4 params
So, we "penalise" the χ^2, use no. degrees of freedom (n.d.f.) = n - #params
It tells us the χ^2 = 9.24 for 8 degrees of freedom (10 data - 2 params = 8 d.f.)
Thus χ^2/n.d.f. = 1.16 not bad!

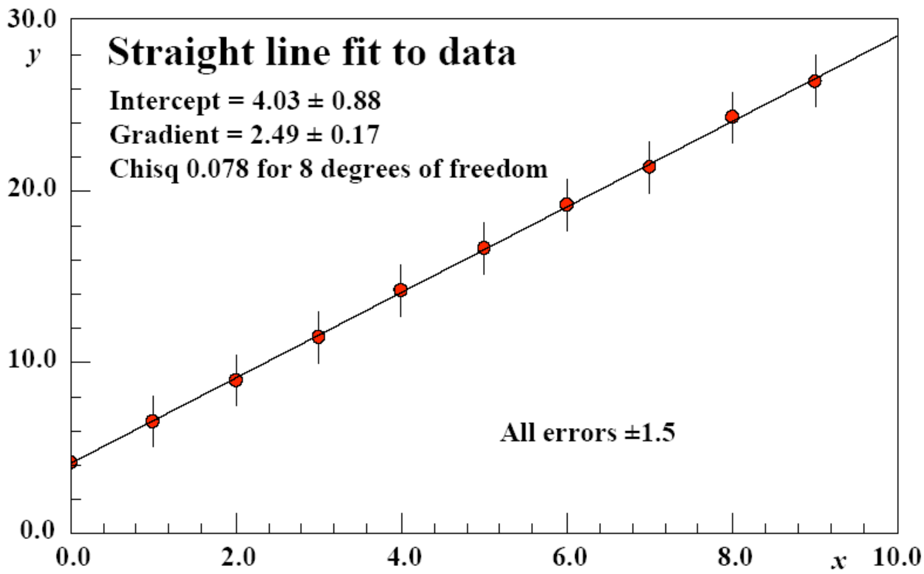


In this example the data scatter more:

similar straight line fit

look at χ^2/n.d.f. = 35.8 / 8 = 4.48

This means that either the model is wrong (i.e. straight line) or errors too small



In this example the data scatter much less:

similar straight line fit

look at χ^2/n.d.f. = 0.078 / 8 = 0.0098

when χ^2/n.d.f. is too small, probably means errors are underestimated