Scientific Measurement

PHY-103

Dr. Eram Rizvi & Dr. Jeanne Wilson

Lecture 3 - Errors & Uncertainties

Queen Mary
University of London

Significant Figures

How many significant figures should we use?

Depends how well we know the error!

For now I state that the uncertainty on the error is never better than 10%

Rule of Thumb:

Look at first 3 sig.figs of the error

quote 2 sig.figs if < 355

quote 1 sig.fig if > 355

Examples:

0.713 ± 0.059 should be given as 0.71 ± 0.06

12.1749 ± 0.0112 should be given as 12.175 ± 0.011

0.07341 ± 0.0462 should be given as 0.73 ± 0.05

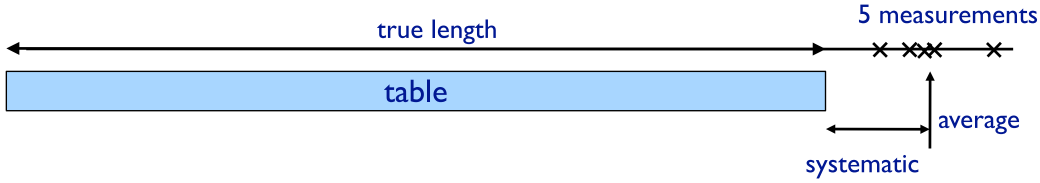
Keep the precision the same for error and central value

i.e. same number of decimal places

e.g. never 123.57 ± 2 !!!

Measurement Uncertainty

Measurement errors usually have 2 components:
- Systematic
- Statistical



Systematic Uncertainties

- Arise from the system or method of measurement itself
- Possible miscalibration errors
- eg: slow running clock
- Stretched ruler
- If you know there is a miscalibration you correct the measurement
- Systematic uncertainties account for possibility of a mistake
- Account for possible error in your assumptions
- These affect your measurements in systematic way: e.g. lengths all too long

Measurement Uncertainty

Random Statistical Uncertainties

- Arise from many small random effects
- Likely to measure too low as well as too high
- eg. parallax errors reading a scale
- vibrations in machinery printing scale on rulers during manufacture
- different reaction times of people in timing experiments
- Random errors dependent on accuracy of equip. (e.g. watch vs atomic clock)
- Affect spread of measurements not average value

We will not discuss systematic errors in any great detail

Concentrate on random errors

Can come back to this at end of course (if you like!)

Averaging Measurements

How best to define average?

Median

Take the middle value:

odd number of measurements: 2.1 2.2 2.3 2.8 5.4

take value in the middle: 2.3

even number of measurements: 2.1 2.2 2.3 2.8 5.4 70.1

take value half way between central measurements: 2.55

Mode

Most frequent value

e.g. 1 5 7 7 8 10 12 15 20

take 7

May not be uniquely defined

e.g. 1 1 2 3 3 5

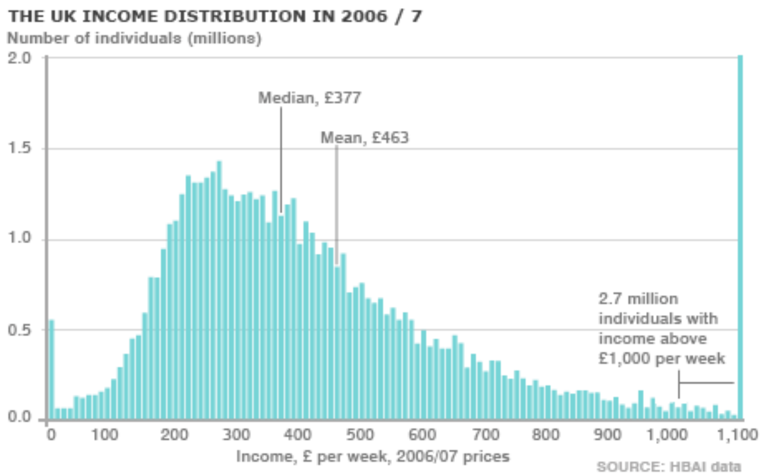
Mean (Arithmetic Mean)

Most familiar case

$$\bar{x} = \langle x \rangle = \frac{1}{N}\sum_i x_i$$

e.g. 2.0 2.1 2.2 2.2 2.2 2.3 2.5

$\langle x \rangle = 2.214$



If all measurements have equal precision $\langle x \rangle$ is best average

Consider

2.1 2.2 2.3 2.4 and 1.4 2.1 2.3 3.2

Both have $\langle x \rangle = 2.25$

One has smaller range than other

Means better precision!

Use this to define the error

How???

a) Range
use x_{min} and x_{max}
Problem: can be 'thrown' by one bad reading

b) Residual
Average distance of each measurement from the mean <x>

$$r = \frac{1}{N}\sum_i (x_i - < x >)$$

consider 2.1 2.2 2.3 2.4 = (-0.15) + (-0.05) + (0.05) + (0.15) = 0
r = 0 !!!

$$\begin{aligned} &\frac{1}{N}\sum (x_i - \bar{x}) \\ &= \frac{1}{N}\sum x_i - \frac{1}{N}\sum \bar{x} \\ &= \frac{1}{N}\sum x_i - \frac{1}{N} N\bar{x} \\ &= 0 \end{aligned}$$

c) Try squaring

Define variance:

$$\sigma^2 = \frac{1}{N}\sum_i (x_i - \bar{x})^2$$

Can be zero only when all deviations are zero
Variance has units of x^2
Define Standard Deviation or Error

$$\sigma = \sqrt{\frac{1}{N}\sum_i (x_i - \bar{x})^2}$$

Standard Properties

Refinement: mathematically can show a better definition is

$$\sigma = \sqrt{\frac{1}{N-1}\sum_i (x_i - \bar{x})^2}$$

Known as Bessel's Correction
Is small for large N
For N=1 Std.Dev. is infinite - cannot define a spread for 1 measurement!

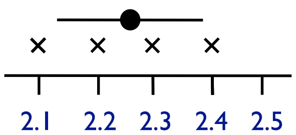
Consider: 2.1 2.2 2.3 2.4
<x> = 2.25

$$\sigma = \sqrt{\frac{1}{4-1}(0.15^2 + 0.05^2 + 0.05^2 + 0.15^2)}$$

$$= \sqrt{\frac{1}{3}(0.0500)} = 0.129$$

= 0.112 without Bessel's correction

Standard Properties



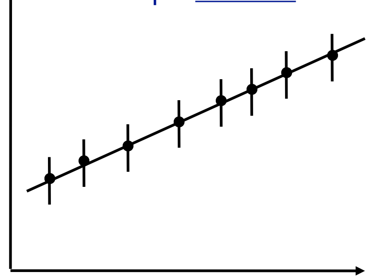
Write measurement as 2.25 ± 0.13 cm
Note number of sig.figs

Some data lie outside of the standard deviation

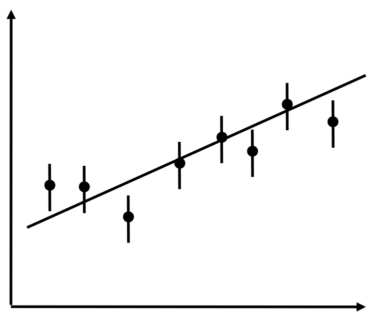
Others are close to the mean <x>

If errors are random expect 2/3 measurements within σ & 1/3 outside σ

For a straight line expect roughly
1/3 data above line
1/3 data below line
1/3 data on the line
This example: statistical errors "too large"



This example: data scatter as we expect
Assuming straight line model is correct



<u>Important point:</u>

σ estimates spread in readings

More data will not change σ greatly

It is governed by precision of measurements

But: more data allows better determination of the mean

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{N}}$$

Note the difference:

σ is the spread of measurements

σ_{mean} is precision of the mean value

σ_{mean} improves with $\sqrt{N}$ i.e. 4 times more data = half the error

Consider age of all ~300 people in physics dept.

Average age is known with good precision

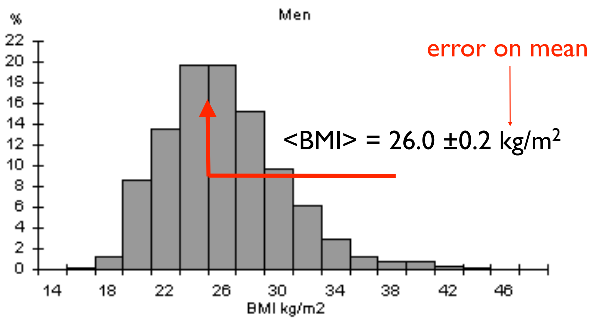
But spread is large (since range is ~18 up to >70!)

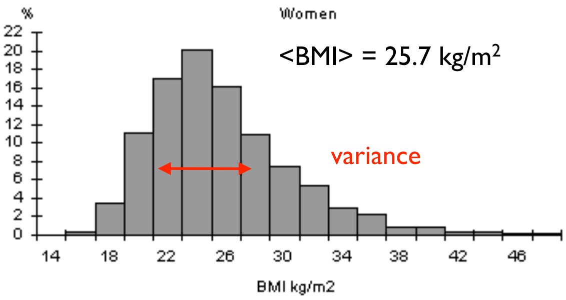
Often used to graphically show statistical data

Abscissa (x) shows measurement value

Ordinate (y) shows number of measurements of frequency of value occurring

Abscissa divided into bins covering range of values





BMI = mass (Kg) / Height^2 (m^2)

mean / variance / error on mean

are properties of <u>any</u> distribution of data

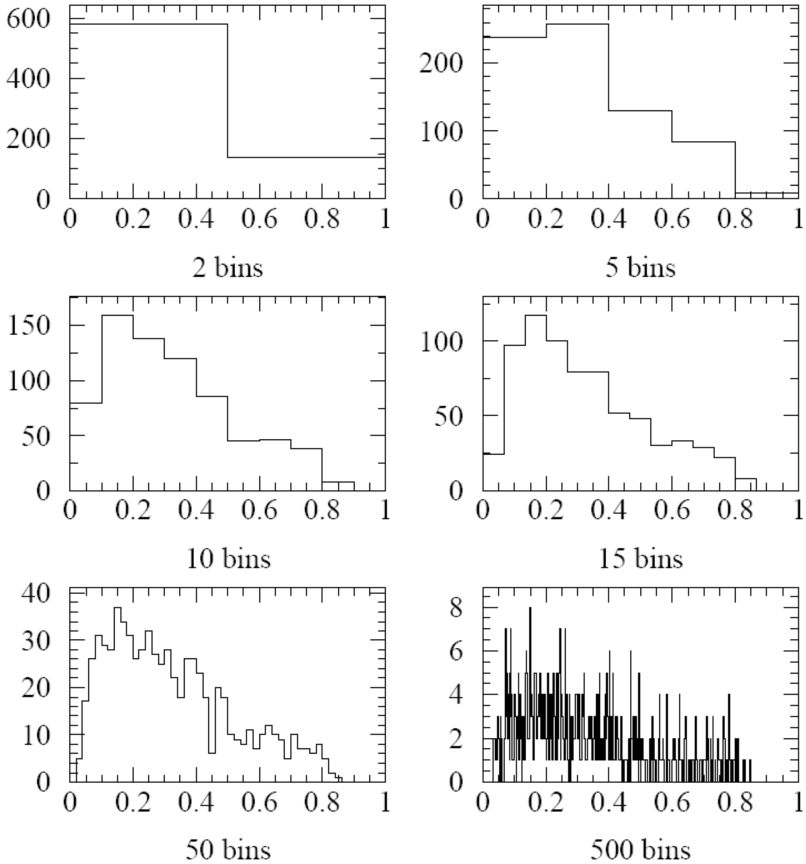
used to characterise the data

convention:

30.000 ....31.9999 in one bin not 32.000

PhysPlot will histogram your data

0.11244
0.19473
0.13776
0.26983
0.12174
0.66705
0.47185
0.44997
0.72706
0.66966
0.47720
0.36061
0.53101
0.25013
0.49582
0.36204
0.56198
0.77228
0.16253
0.25293
0.62261
0.55745
0.23380



Same data shown with different binning

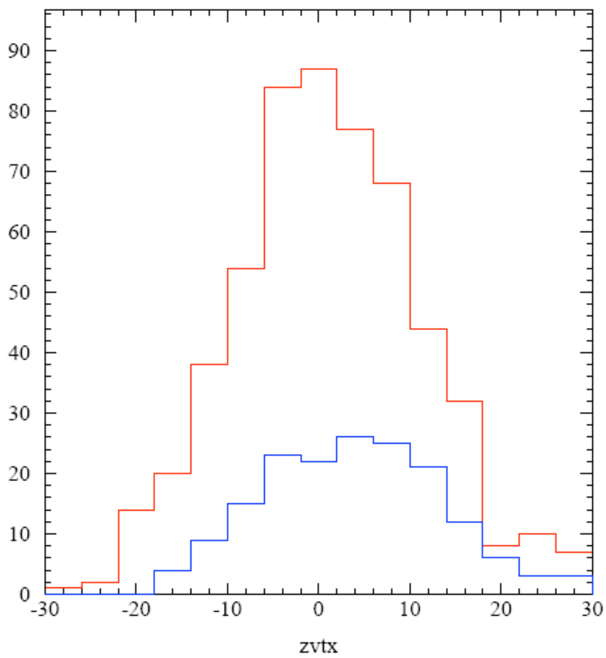
Choose binning carefully

Aim to show smooth distribution/structure

Too wide: lose info

Too narrow: large statistical fluctuations

How do we compare histograms with different amount of data?



We can calculate the mean, variance

Can also normalise data

Divide contents by total no. measurements

before normalising

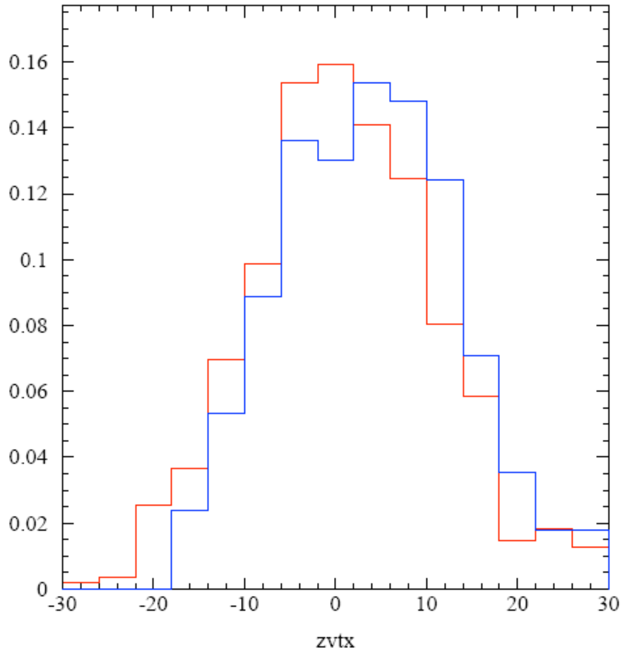
$$\sum \text{bins} = 550$$

after normalising

$$\sum \frac{\text{bins}}{N} = \frac{N}{N} = 1$$

area of histogram = 1

normalised histograms



blue histogram is shifted to right with respect to red histogram

histograms are not the same!

could also compare the mean values and error on the mean

Sometimes you might wish to know the asymmetry of our data
calculate the Skewness

$$\text{skewness} = \frac{\text{mean} - \text{mode}}{\sigma}$$

Note: dimensionless quantity
Skewness = 0 for symmetric distribution

We have defined several statistical properties of an arbitrary set of data:

$$\frac{1}{N}\sum_i x_i$$

mean

$$\sigma^2 = \frac{1}{N}\sum_i (x_i - \bar{x})^2$$

variance

$$\sigma = \sqrt{\frac{1}{N}\sum_i (x_i - \bar{x})^2}$$

standard deviation

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{N}}$$

error on the mean

$$\sum \frac{\text{bins}}{N} = \frac{N}{N} = 1$$

Normalisation conditon

skewness $= \dfrac{\text{mean} - \text{mode}}{\sigma}$

Random/statistical uncertainties affect variance of measurements

Random errors do not affect mean of measurements

Systematic uncertainties affect the mean of measurements

Error on the mean improves as $1/\sqrt{N}$