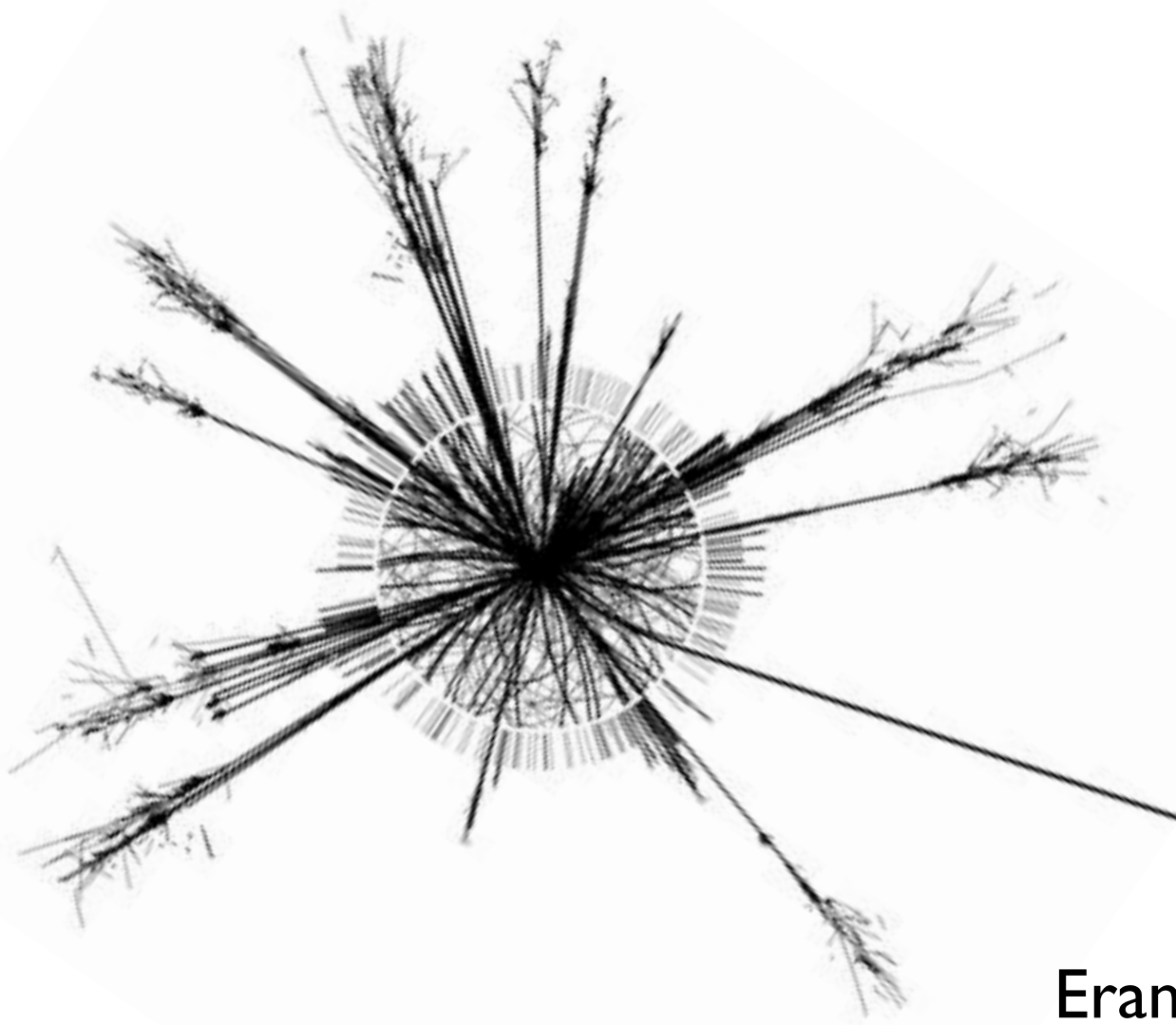


# The Standard Model of Particle Physics - I

## Lecture 3

- Quantum Numbers and Spin
- Symmetries and Conservation Principles
- Weak Interactions
- Accelerators and Facilities



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## **A Century of Particle Scattering 1911 - 2011**

- scales and units
- overview of periodic table → atomic theory
- Rutherford scattering → birth of particle physics
- quantum mechanics - a quick overview
- particle physics and the Big Bang

## **A Particle Physicist's World - The Exchange Model**

- quantum particles
- particle detectors
- the exchange model
- Feynman diagrams

## **The Standard Model of Particle Physics - I**

- quantum numbers
- spin statistics
- symmetries and conservation principles
- the weak interaction
- particle accelerators

## **The Standard Model of Particle Physics - II**

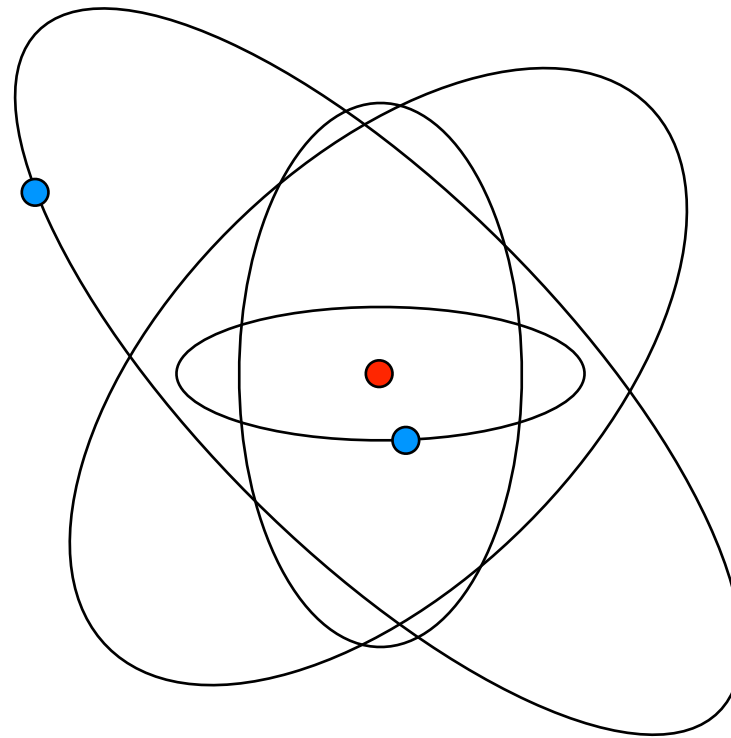
- perturbation theory & gauge theory
- QCD and QED successes of the SM
- neutrino sector of the SM

## **Beyond the Standard Model**

- where the SM fails
- the Higgs boson
- the hierarchy problem
- supersymmetry

## **The Energy Frontier**

- large extra dimensions
- selected new results
- future experiments



Why do electrons not fall into lowest energy atomic orbital?

Niels Bohr's atomic model:

angular momentum is quantised  $\Rightarrow$  only discrete orbitals allowed

Why do electrons not collapse into low energy state?

What do we mean by 'electron' ?

charge =  $-1$

spin =  $1/2$

mass =  $0.511 \text{ MeV}/c^2$

Any object satisfying these conditions is an electron!

True for all particles

A set of quantum numbers specifies the particle type

Quantum numbers are encoded in the wave function

Wave function  $\psi$  is related to probability of observing the particle

Quantum particles possess spin

Measured in units of  $\hbar = h/2\pi$

All particles fall into 2 categories

Fermions:

spin =  $1/2, 3/2, 5/2 \dots$

Bosons:

spin =  $0, 1, 2 \dots$

P = Probability of observation

A = some constants of nature

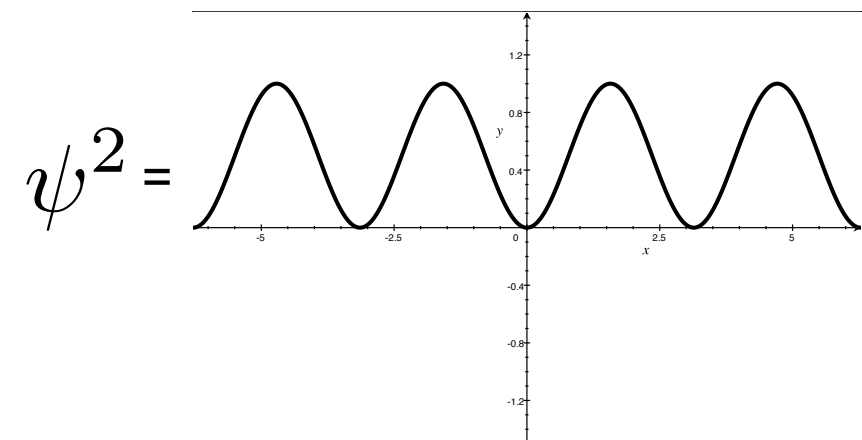
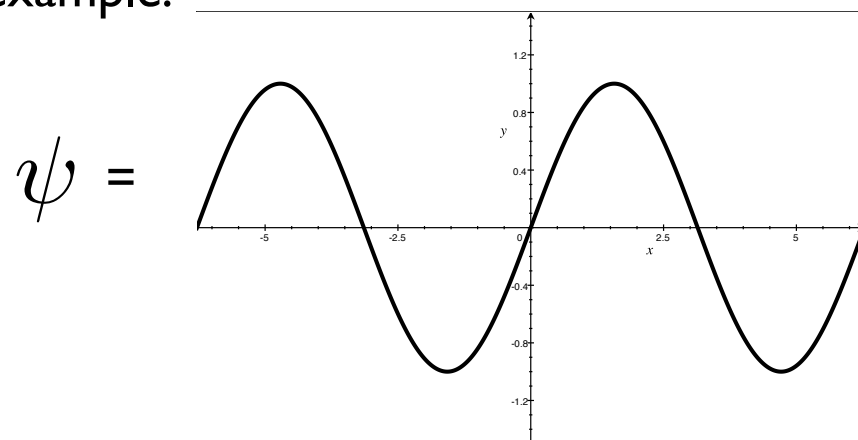
$$P = A\psi^2$$

probability can never be negative

probability can never be  $> 1$

Experiment is only sensitive to  $\psi^2$

For example:

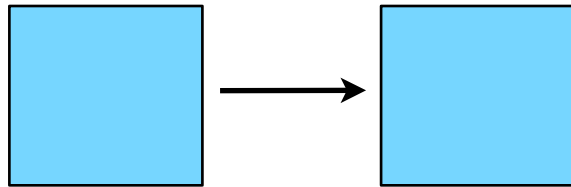




Many forces and phenomena in nature exhibit symmetries

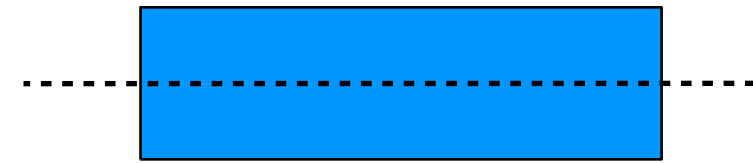
An object has a symmetry if an operation/transformation leaves it invariant (i.e. the same)

## Rotational Symmetry



Squares rotated  $90^\circ$  remain unchanged  
transformation is  $\theta \rightarrow \theta + 90^\circ$

## Reflection Symmetry = Parity



Rectangle reflected about axis is invariant  
transformation is  $x \rightarrow -x$

## Mass symmetry

$$F = G \frac{m_1 m_2}{r^2} = G \frac{m_2 m_1}{r^2}$$

Newtons Law is symmetric about  
transformations of  $m_1$  and  $m_2$

## Time symmetry

Physics remains the same if time runs backwards  
Reflections about the  $t=0$  axis are invariant  
transformation is  $t \rightarrow -t$

Parity is simply a reflection about a given axis in space

In 3d we reflect object about all three Cartesian axes

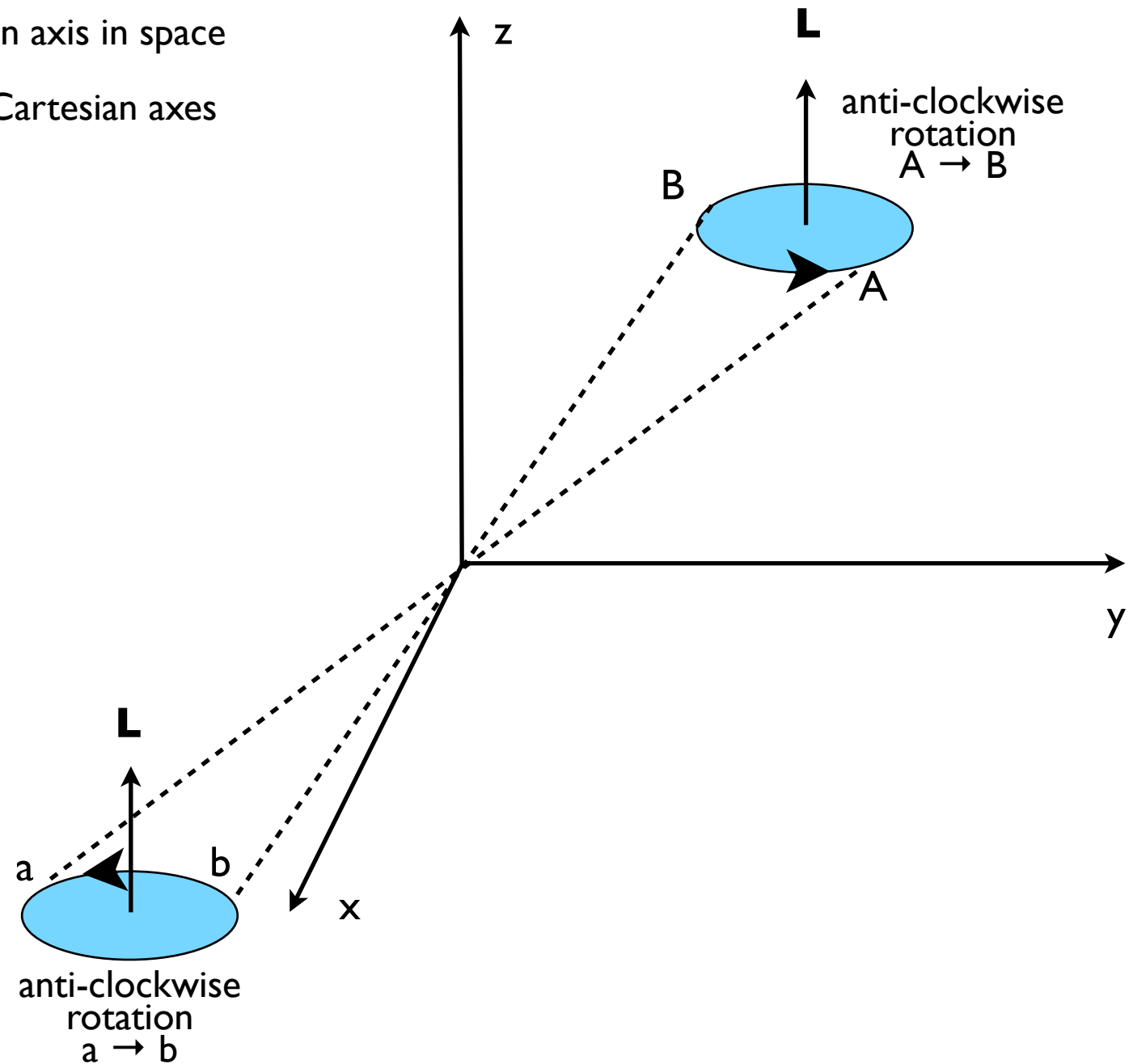
$$x \rightarrow -x$$

$$y \rightarrow -y$$

$$z \rightarrow -z$$

$$A \rightarrow a$$

$$B \rightarrow b$$



Some quantities do not flip sign:

**L** = spin or angular momentum

This remains the same after a parity inversion

In terms of wave functions of a particle trapped in a “box” by a potential energy field  $V$

$V = 0$  inside the box

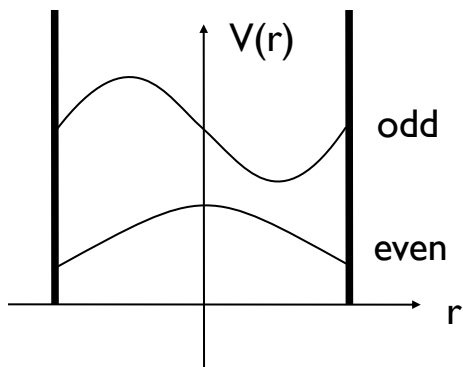
$V = \infty$  at the walls of the box  $\Rightarrow$  particle is trapped

we solve Schrödinger equation to find all allowed states  $\square$

$$\frac{-\hbar^2}{2m} \nabla^2 \psi + V(r) \psi = i\hbar \frac{\partial}{\partial t} \psi$$

If potential  $V$  is symmetric:

$$V(r) = V(-r) \quad \text{then: } |\psi(r)|^2 \equiv |\psi(-r)|^2 \rightarrow \psi(-r) = \pm \psi(r)$$



If  $V(r)$  is unchanged then resulting allowed particle states must be even or odd parity

$$\psi(-r) = +\psi(r) \quad \text{even parity}$$

$$\psi(-r) = -\psi(r) \quad \text{odd parity}$$

$$V(r) \neq V(-r) \quad \text{then} \quad |\psi(r)|^2 \neq |\psi(-r)|^2$$

# Two-particle Wave functions



Consider 2 electrons in Helium atom at positions  $r_1$  and  $r_2$  and in different states  $\psi_A$  and  $\psi_B$

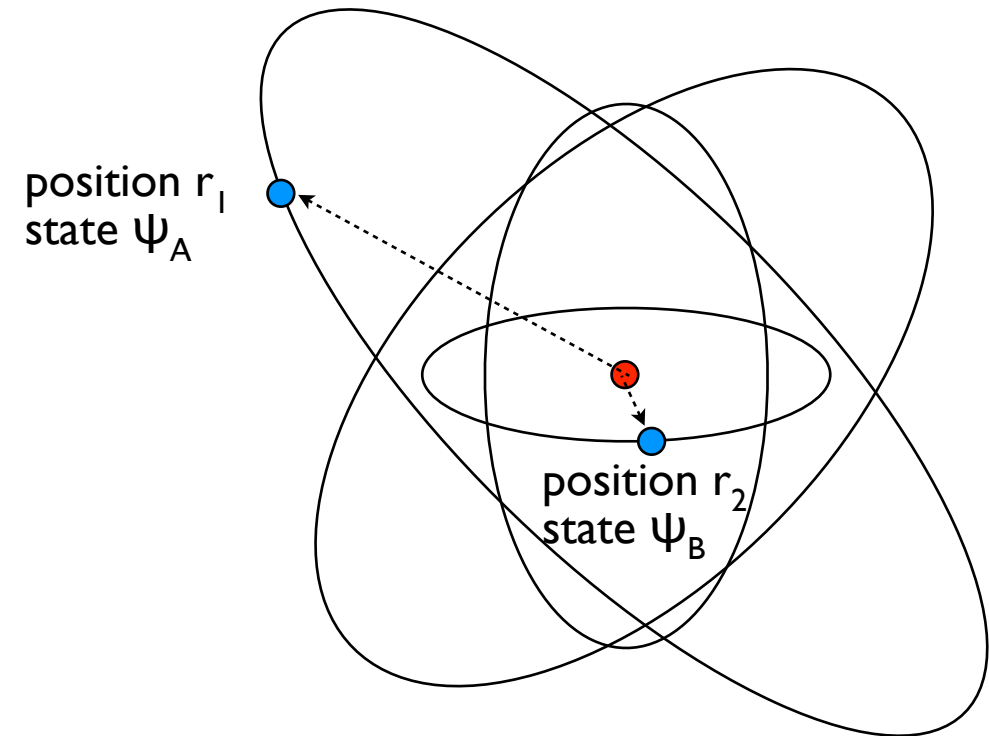
What is the combined two-particle wave function ?

We try:

$$\psi_{AB} = \psi_A(r_1) \cdot \psi_B(r_2)$$

Interchanging the two electrons we get:

$$\psi_{BA} = \psi_B(r_1) \cdot \psi_A(r_2)$$



Electrons are indistinguishable - they do not carry labels to identify one from another

All electrons are the same

So measurement cannot distinguish if two electrons swapped position

But, if a measurement could detect the interchange then they are not **indistinguishable**

Therefore this 2-particle combined wave function is no good!



# The Pauli Exclusion Principle



For indistinguishable particles, probability densities must be invariant to exchange i.e.  $\psi_{AB}$  can differ only by sign from  $\psi_{BA}$

Wave-functions are symmetric if  $\psi_{AB} = +\psi_{BA}$

Wave-functions are anti-symmetric if  $\psi_{AB} = -\psi_{BA}$

Experiments show:

particles with integer spin (0,1,2...) have symmetric wave-functions

particles with half integer spin ( $1/2, 3/2, 5/2$ ) have anti-symmetric wave-functions

For two particle systems take wave function of the form:

$$\psi_{AB} \simeq [\psi_A(r_1)\psi_B(r_2) \pm \psi_B(r_1)\psi_A(r_2)]$$

bosons : integer spin    +  
fermions: half integer spin    -

Wolfgang Pauli



Nobel Laureate 1945

What happens for two identical particles in same quantum state?  $\psi_A = \psi_B$

$\psi_{AB} \rightarrow 0$  for identical fermions (half integer spin particles)

Pauli Exclusion Principle: Identical fermions cannot occupy the same quantum state!

This stops the atom from collapse!

Consider this function  $f(x)$ :

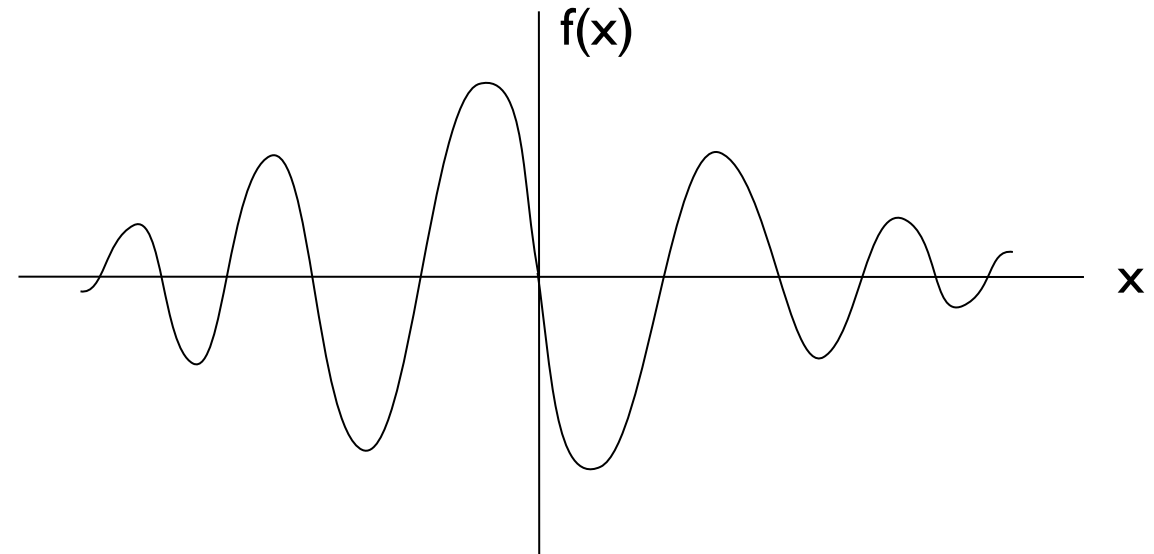
exact functional form is unknown

What do we know about it?

it is odd i.e.  $f(-x) = -f(x)$

Fourier transform will have no cos terms

sum from  $-n$  to  $+n$  is 0



So symmetries can provide useful information

Particularly dynamical symmetries of motion

Greeks believed stars moved in circles since they were most symmetrical orbits

Newton realised fundamental symmetries revealed **not** in motions of individual objects...

...but in **set of all possible motions**

Symmetries are manifest in equations of motion - not particular solutions to those equations

Gravity is spherically symmetric, but planets have elliptical orbits

In 1917 role of symmetries in physics was understood: Emmy Noether's Theorem:

Every symmetry of nature yields a conservation law (and vice versa!)

Translation in time	Energy conservation
Translation in space	Momentum conservation
Rotations	Angular Momentum conservation
Gauge Transformation	Charge conservation

Laws of physics are symmetric with respect to translations in time:  
they are the same today as they were yesterday

Analysing the set of transformations of a system uses Group Theory

Imagine swapping all numbers +ve and -ve e.g. my bank account  
Doing this does not break the laws of mathematics  
I suddenly become very rich indeed...

...but if the entire world did this simultaneously we would notice no difference at all!  
The banking world is symmetric under sign inversion of real numbers  
The total amount of money is conserved !!

A symmetry principle has yielded a law of conservation

In forming theories physicists look for symmetries to help explain phenomena  
In particle physics such symmetries tell us of the nature of space-time & particles themselves  
Consider charge symmetry: **replace all particles with anti-particles**

Our universe is composed of matter

We **happen** to call it “matter”

We **could** have called it “anti-matter”

But, we **do know** that we are made of the same type (either all matter or not)

Fundamental laws of nature do not tell us if we are made of matter or anti-matter

There is no difference!

Same is true of parity inversion:

Parity inversion turns left-handed co-ordinate system into a right-handed one

There is no fundamental distinction between “left” and “right”

OK, humans have hearts on left side of body...

But we concern ourselves with laws of nature, not accidental arrangement of objects

Similarly for time reversal symmetry:

If we replace  $t$  with  $-t$  in all equations

If we did turn time backwards - laws of physics are expected to be the same

In this universe time does run forwards...

...if we changed this, we don't expect Newton's laws to fail

At least, we expect universe to behave this way...

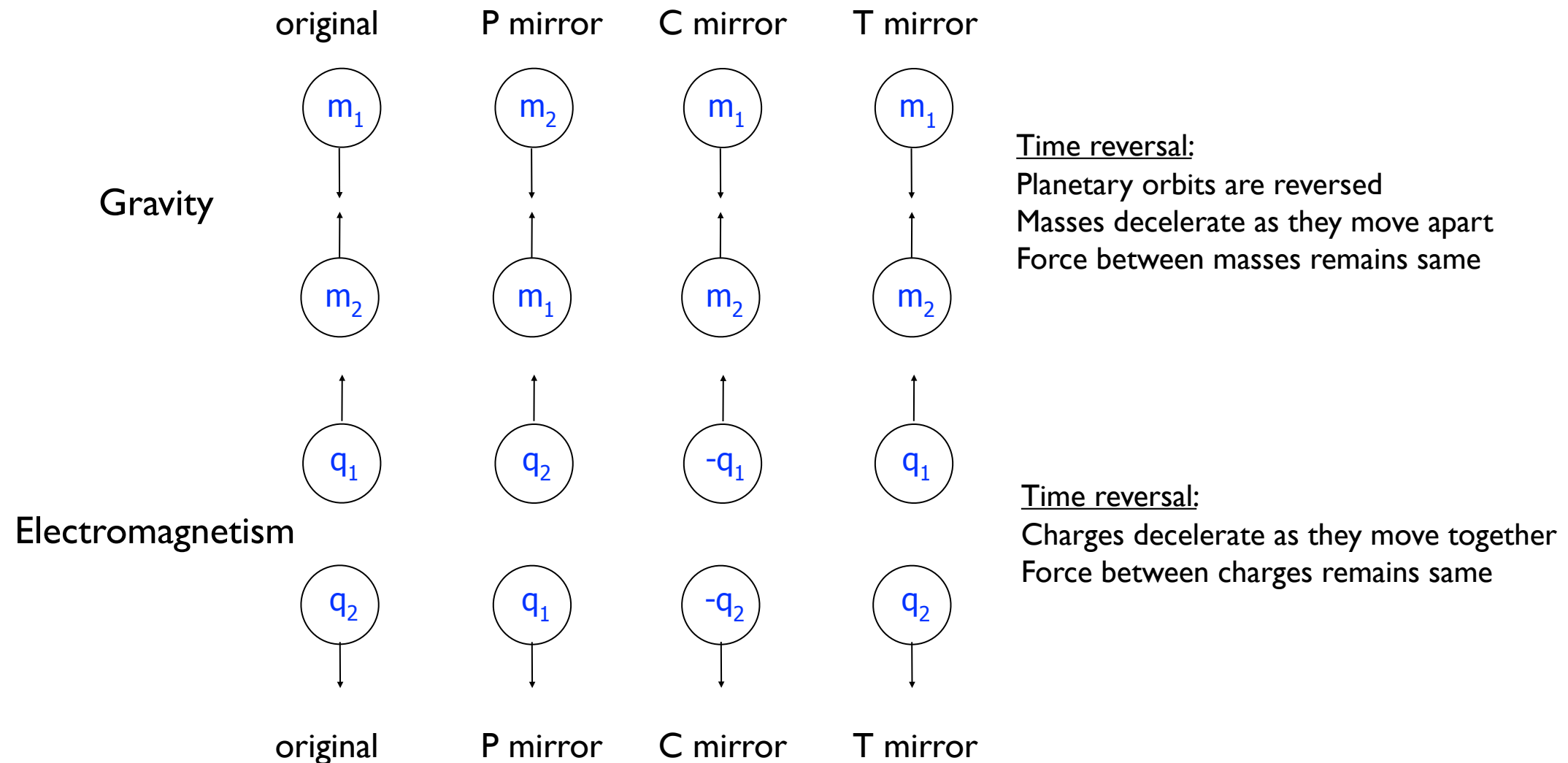


Charge, Parity & Time are 3 fundamental symmetries:

C	Charge conjugation	swap all particles with anti-particles
P	Parity inversion	reflect system about origin: $P\psi(r) = \psi(-r)$
T	Time reversal	time is reversed in direction: $T\psi(t) = \psi(-t)$

Gravity, & electromagnetism are both invariant to C, P and T inversions

8 experiments of two particles  
with masses  $m_1$  and  $m_2$  and  
electric charge  $q_1$  and  $q_2$

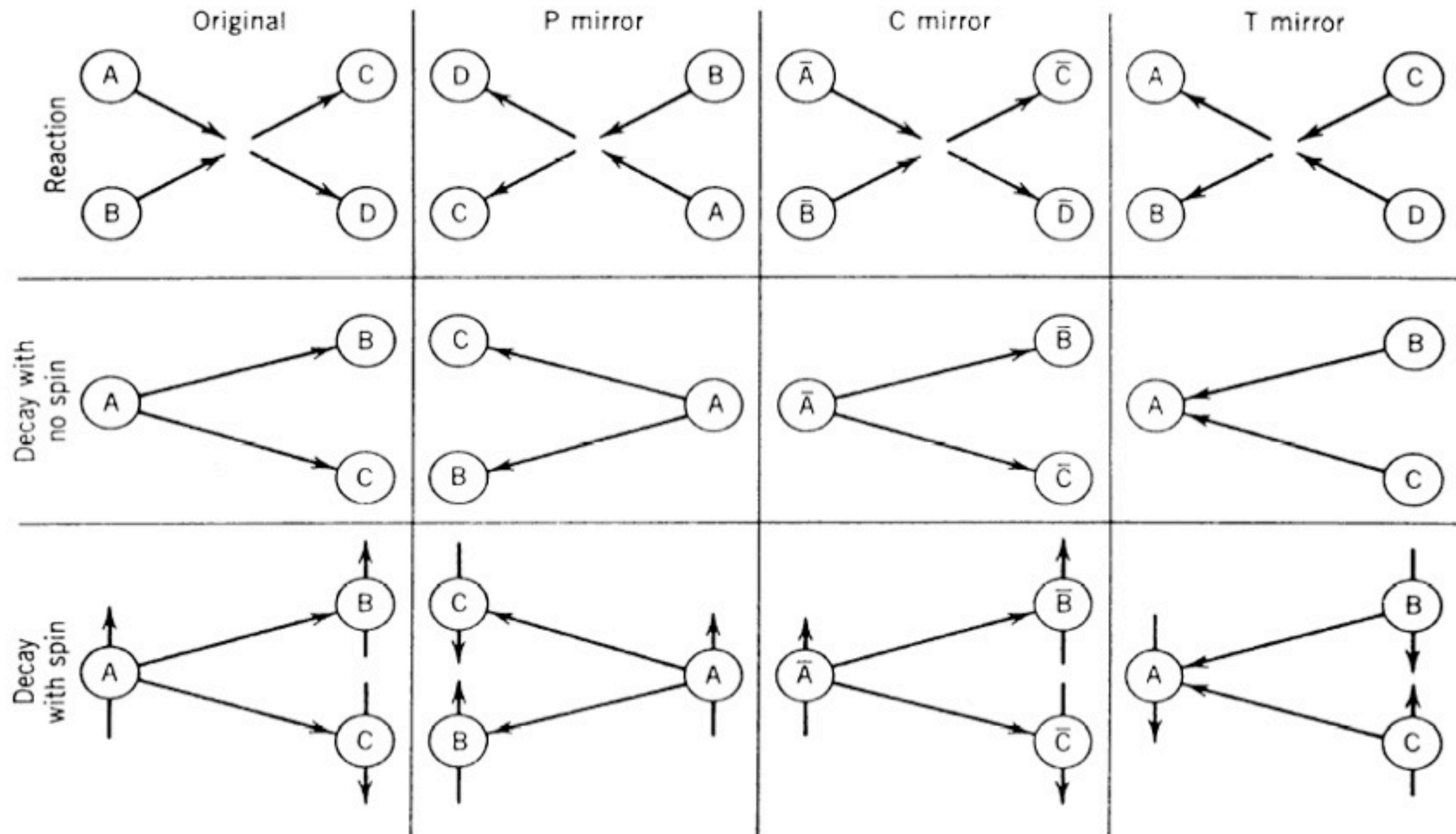


Can test C, P, and T symmetries in a series of experiments:  $A+B \rightarrow C+D$

**Test P:** swap positions of A and B (i.e. projectile A and target B, instead of vice versa)

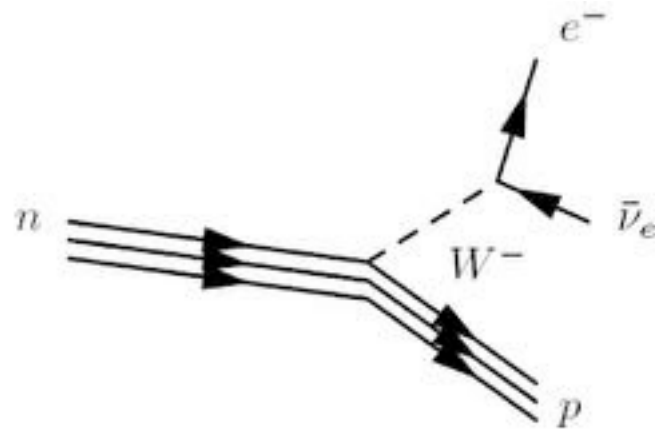
**Test C:** exchange A and B for anti-particles

**Test T:** collide C and D to produce A and B



**Wolfgang Pauli** wrote: *“I do not believe that the Lord is a weak left-hander, and I am ready to bet a very large sum that the experiments will give symmetric results”*

**Richard Feynman** bet \$50 that experiment would confirm parity as a valid symmetry



Beta decay is a weak process  
Mediated by W exchange

Madame Wu



In 1956 Chien-Shiung Wu showed parity is violated in radioactive beta decay  
Weak force does not conserve parity!

	<b>Conserves Charge?</b>	<b>Conserves Parity?</b>
<b>Electromagnetism</b>	yes	yes
<b>Strong</b>	yes	yes
<b>Weak</b>	yes	no!

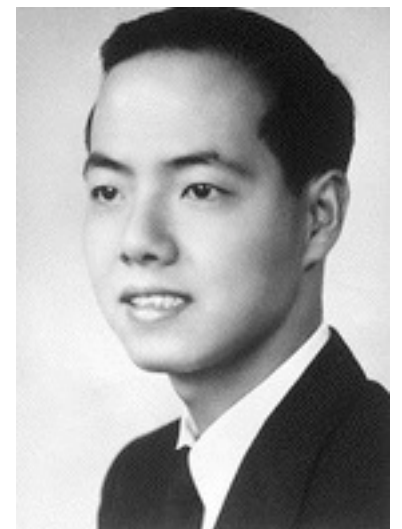
Laws of physics are different for real-world and mirror experiments!

Lee & Yang predicted parity violation in 1956  
Awarded Nobel prize in 1957 after Wu's validation

Nobel Laureates 1957

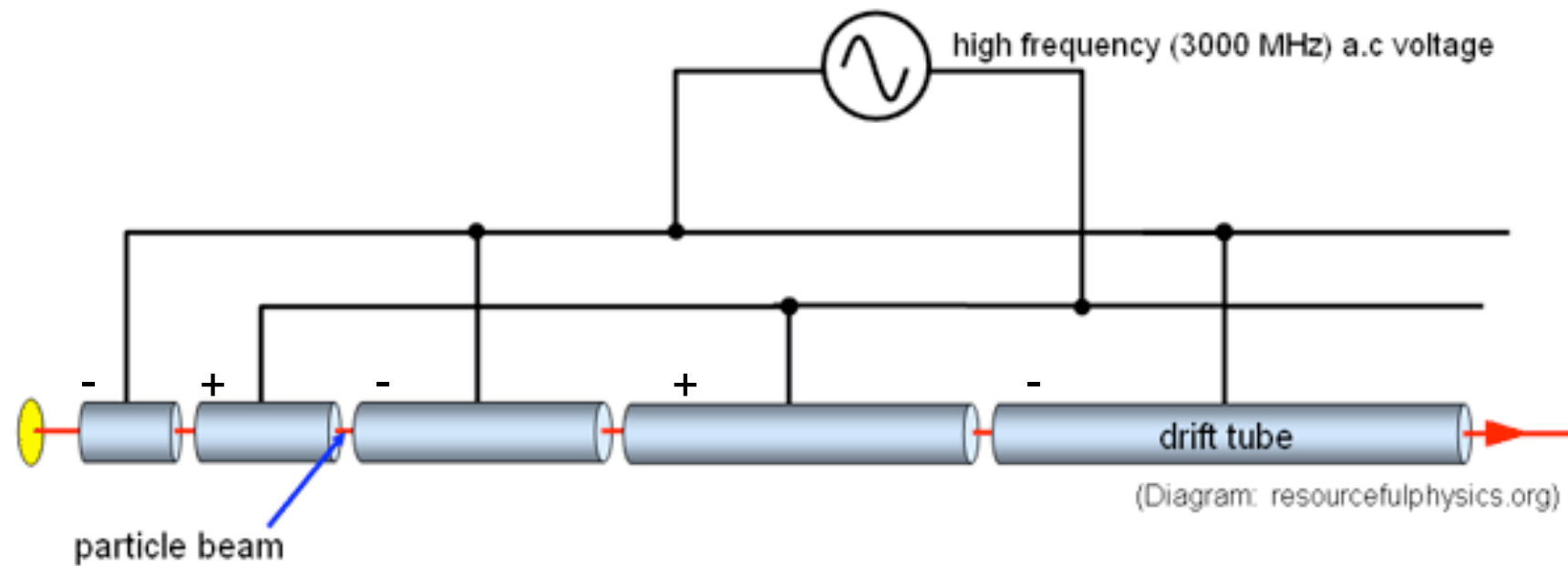


C.N. Yang

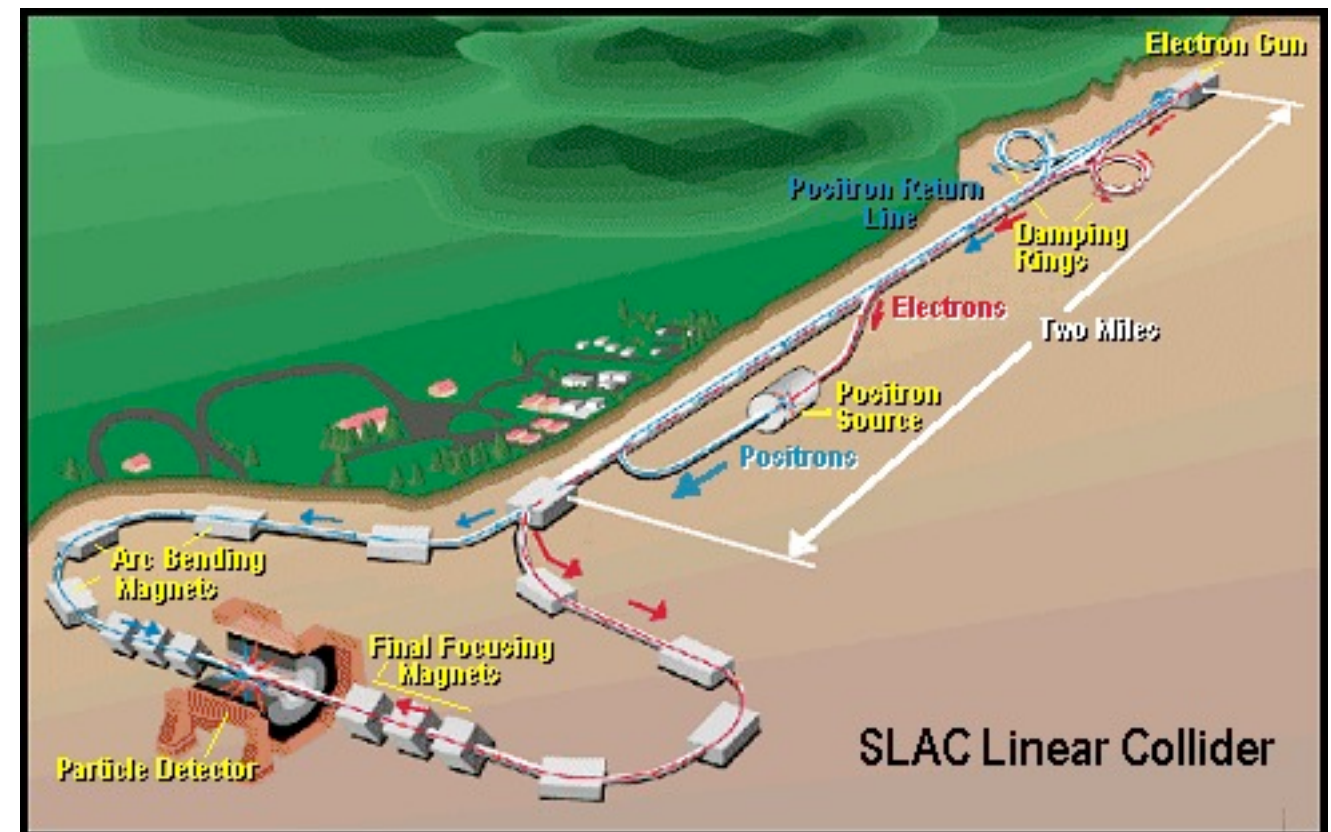


T.D. Lee





No electric field inside drift tubes  
Electric field non-zero between tubes  
Adjust tube gap as particle accelerates

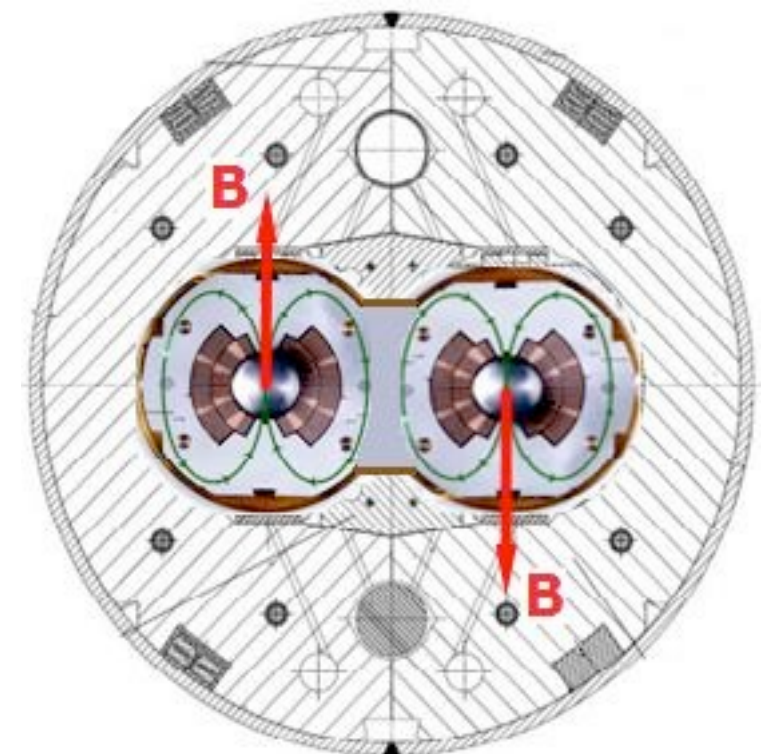




## The Large Hadron Collider



27 km circumference tunnel in France / Switzerland - near Geneva  
 Highest energy accelerator in the world  
 Protons accelerated to 7,000 GeV = 99.9999991% speed of light  
 High vacuum  
 Super cold superconducting magnets to achieve strong magnetic fields  
 17,000 A current in magnets  
 Four experiments:  
     Atlas  
     CMS  
     LHCb  
     Alice





Operating temperature:  $-271^{\circ}\text{C}$  One of the coldest places in universe  
High energy collisions equivalent to temperatures 100,000 times hotter than sun's core  
High vacuum needed to avoid unwanted collisions with air molecules - less dense than solar system  
1200 dipole magnets to bend the protons  
Protons circulate 11,000 times per second  
Generates up to 600 million collisions per second  
LHC costs for material, construction, personnel (excluding experiments) = € 3,000,000,000

