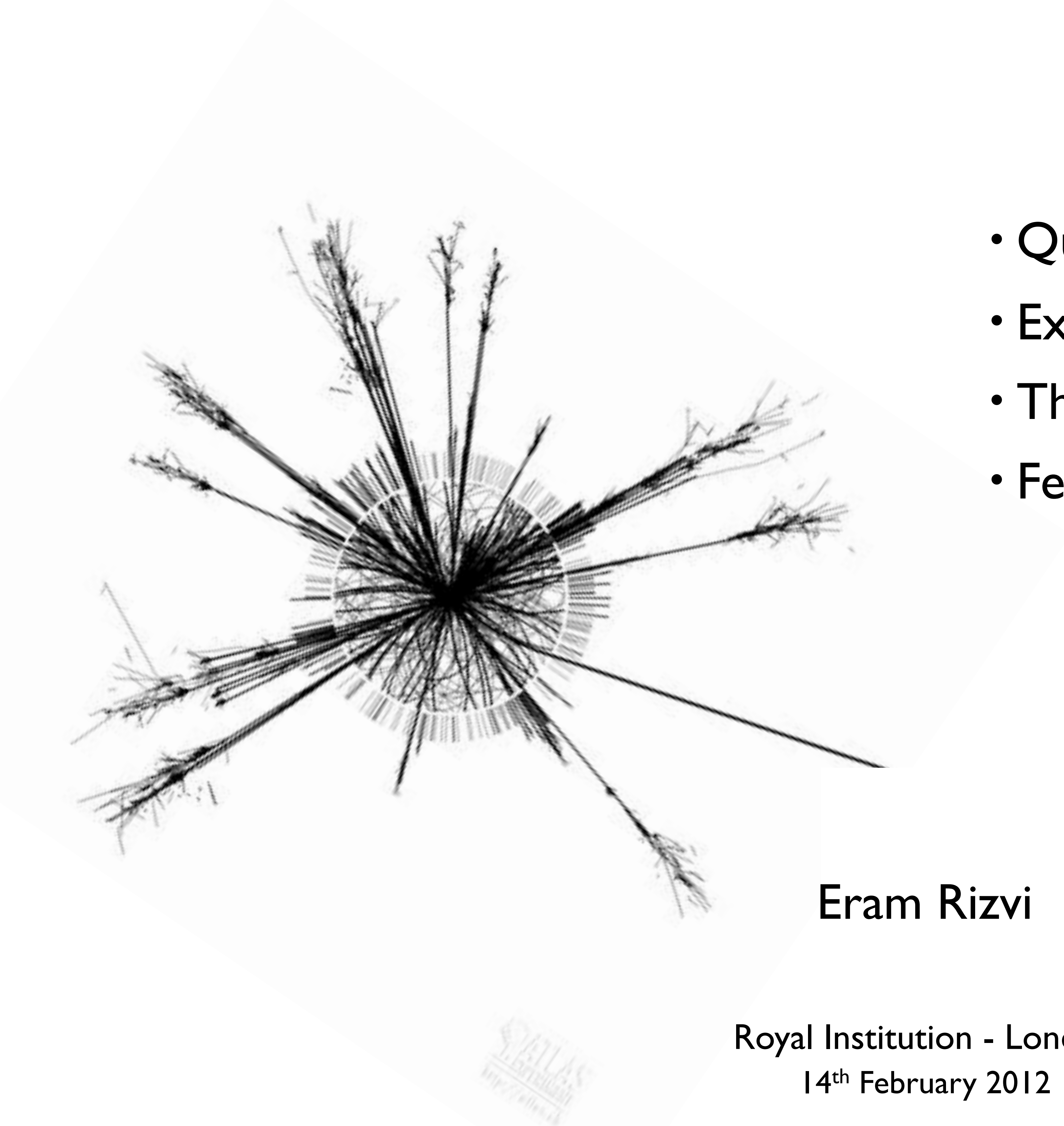
The Exchange Model



Lecture 2

- Quantum Particles
- Experimental Signatures
- The Exchange Model
- Feynman Diagrams

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Outline

A Century of Particle Scattering 1911 - 2011
- scales and units
- overview of periodic table → atomic theory
- Rutherford scattering → birth of particle physics
- quantum mechanics - a quick overview
- particle physics and the Big Bang

A Particle Physicist's World - The Exchange Model
- quantum particles
- particle detectors
- the exchange model
- Feynman diagrams

The Standard Model of Particle Physics - I
- quantum numbers
- spin statistics
- symmetries and conservation principles
- the weak interaction
- particle accelerators

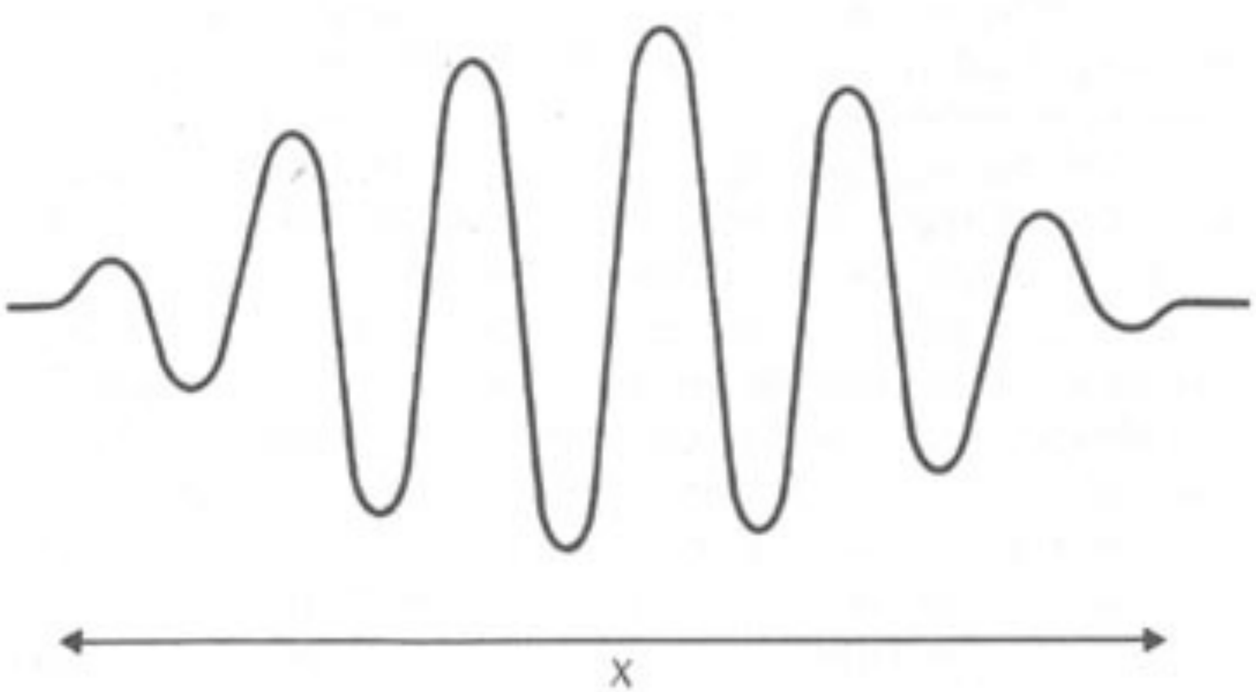
The Standard Model of Particle Physics - II
- perturbation theory & gauge theory
- QCD and QED successes of the SM
- neutrino sector of the SM

Beyond the Standard Model
- where the SM fails
- the Higgs boson
- the hierarchy problem
- supersymmetry

The Energy Frontier
- large extra dimensions
- selected new results
- future experiments

If a ‘particle’ has an associated wave - where is it?
If particle has a single definite momentum it is represented by a single sine wave with fixed λ
But - wave is spread out in space - cannot be localised to a single point

Particle with less well defined energy: i.e. a very very narrow range of momentum Δp
$\Rightarrow$ several sine waves are used to describe it
They interfere to produce a more localised wave packet confined to a region Δx
The particle’s position is known better at the expense of knowing its momentum!



a wave packet corresponding to a particle located somewhere in the region X

This is the origin of the Heisenberg Uncertainty Principle

$$\Delta p \Delta x > h$$

The quantum world is fuzzy!
Cannot know precisely the position and momentum
The trade-off is set by Planck’s constant *h*
h is small $\Rightarrow$ quantum effects limited to sub-atomic world

$h = 4.135 \times 10^{-15}$ eV.s

Wave functions and Operators

Macroscopic objects also have associated wave functions etc
But wavelength is immeasurably small!

$$\lambda = \frac{h}{p}$$

How is information about the particle 'encoded' in the wave function?
The wave function describes and contains all properties of the particle - denoted ψ

All measurable quantities are represented by a mathematical "operator" acting on the wave function

A travelling wave moving in space and time with definite momentum (fixed wavelength/frequency) can be written as:

$$\psi = A \sin\left(\frac{2\pi x}{\lambda} - \omega t\right)$$

A = amplitude of the wave
ω = frequency
λ = wavelength

We choose a position in space , x, and a time t and calculate the value of the wave function

Can also write this in the form: $\psi = Ae^{i(kx-\omega t)}$ and $k = \frac{2\pi}{\lambda}$ (ignore i for now)

If this represents the wave function of particle of definite (fixed) energy E then a measurement of energy should give us the answer E

Wave functions and Operators

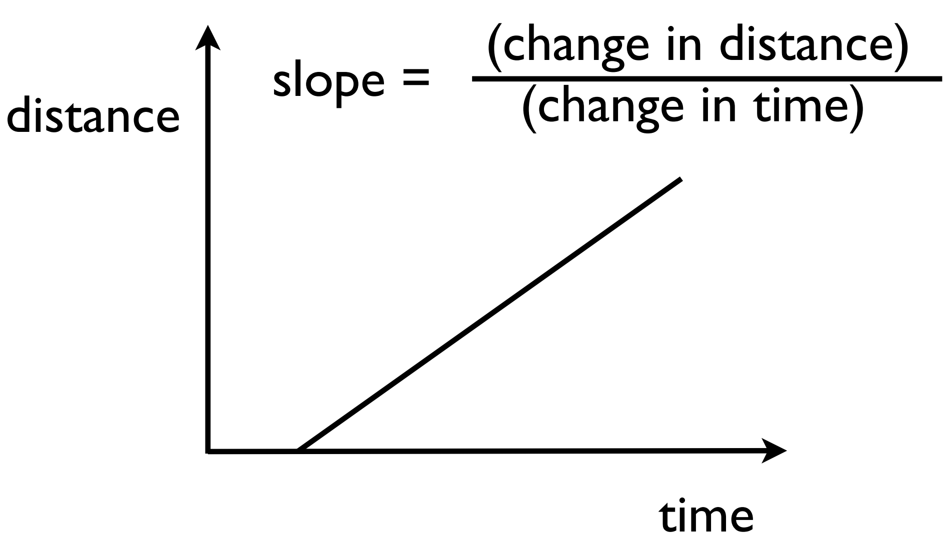
We now posit that all measurements are represented by an operator acting on the wave function
Which mathematical operation will yield the answer E for the particle energy?

$$i\hbar\frac{\partial}{\partial t}$$

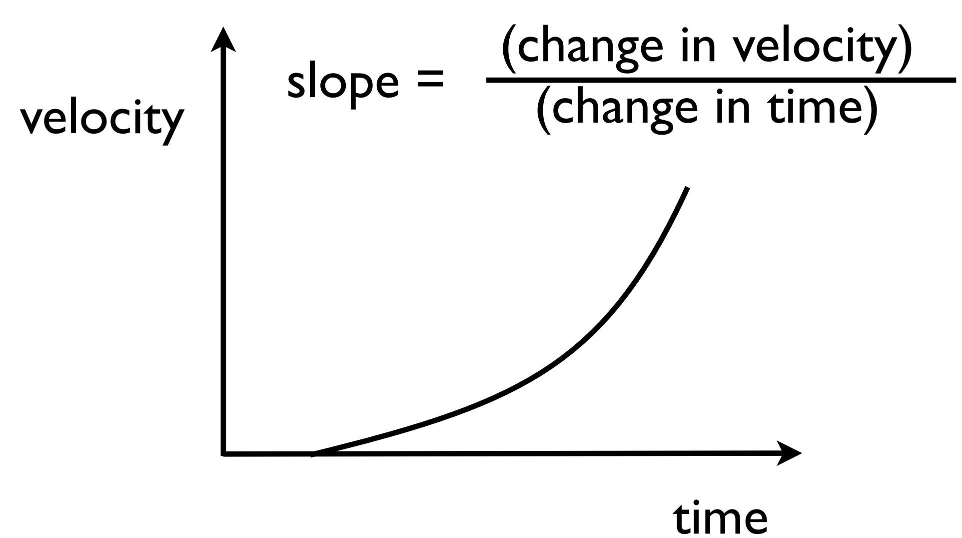
this is the derivative with respect to time = rate of change of something
a derivative calculates the slope of a mathematical function
this is incomplete - it needs something to act on
just like + is incomplete without x and y to act on i.e. x+y
it acts on the wave function ψ

like a verb without a noun to act on

An object's velocity is the
rate of change of object's distance



An object's acceleration is
rate of change of object's velocity



Wave functions and Operators

For a particle with wave function and definite energy E then:

$$i\hbar\frac{\partial}{\partial t}\psi = E\psi$$

This wave notation makes derivatives easier to calculate

$$\psi = Ae^{i(kx-\omega t)}$$

Similarly measurement of momentum for a particle with definite momentum p_x has the operator equation:

$$\frac{\hbar}{i}\frac{\partial}{\partial x}\psi = p_x\psi$$

In both cases the operator leaves the wave function unchanged
It is just multiplied by the momentum, or energy

(mathematically E and p are the eigenvalues of the equation)

Imagine a free particle moving in space - no forces acting on it
Kinetic energy = total energy for the particle

classical equation $$E = \frac{p^2}{2m}$$ but in quantum mechanics $$p \rightarrow \frac{\hbar}{i}\frac{\partial}{\partial x}$$

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = E\psi$$

operator for total kinetic energy
(kinetic energy is energy due to motion)
m = particle mass

For a free particle moving in 1 dimension with no forces acting on it and with definite energy:

1d Schrödinger equation
co-ordinate position x

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = i\hbar\frac{\partial}{\partial t}\psi$$

Notice:
derivatives with respect to spatial co-ordinates are related to momenta
derivatives with respect to time co-ordinate is related to energy

The Schrödinger Equation

In three dimensions (co-ordinate positions x,y,z):

$$\frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi = i\hbar\frac{\partial}{\partial t}\psi \longrightarrow \frac{-\hbar^2}{2m}\nabla^2\psi = i\hbar\frac{\partial}{\partial t}\psi$$

shorthand

Finally we include an interaction of the particle with an external (potential) energy field V

$$\frac{-\hbar^2}{2m}\nabla^2\psi + V(x,y,z)\psi = i\hbar\frac{\partial}{\partial t}\psi$$

this equation can now predict how particle moves / scatters under influence of the field *V(x,y,z)*

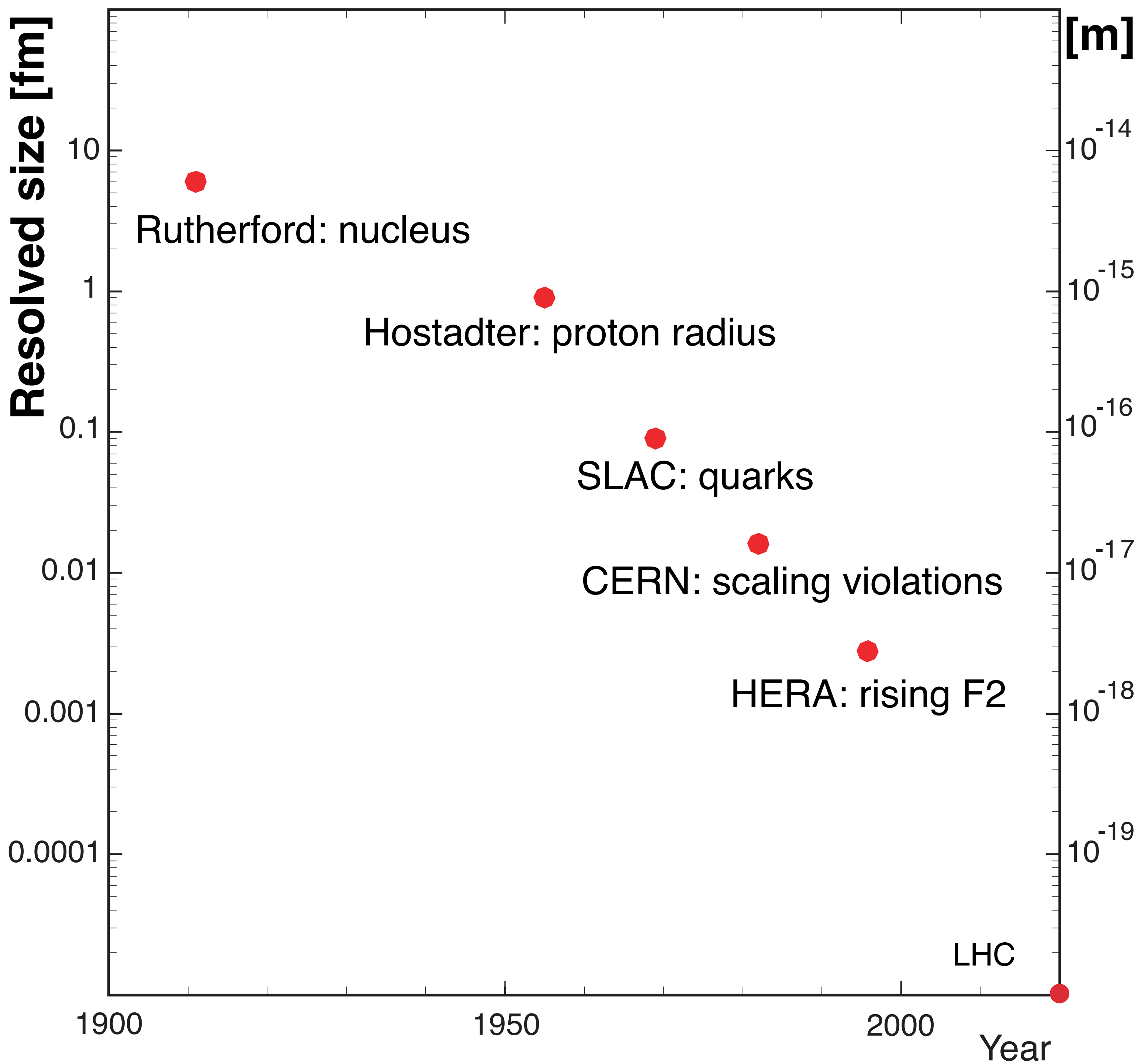
ψ contains all info about our particle
Equation describes how wave function changes momentum and energy when interacting with an energy field V

<u>Rule for Quantum Mechanics:</u>
Use classical equations of physics
Replace all "observables" with quantum operators acting on the particle wave function
Observables = energy, momentum, angular momentum - anything we can measure

(we have neglected relativity though - beyond our scope here)
We will return to this later...

Scattering Experiments

logarithmic scale: 6 orders of magnitude!



publication date of experiment

$$\lambda \simeq \frac{hc}{E}$$

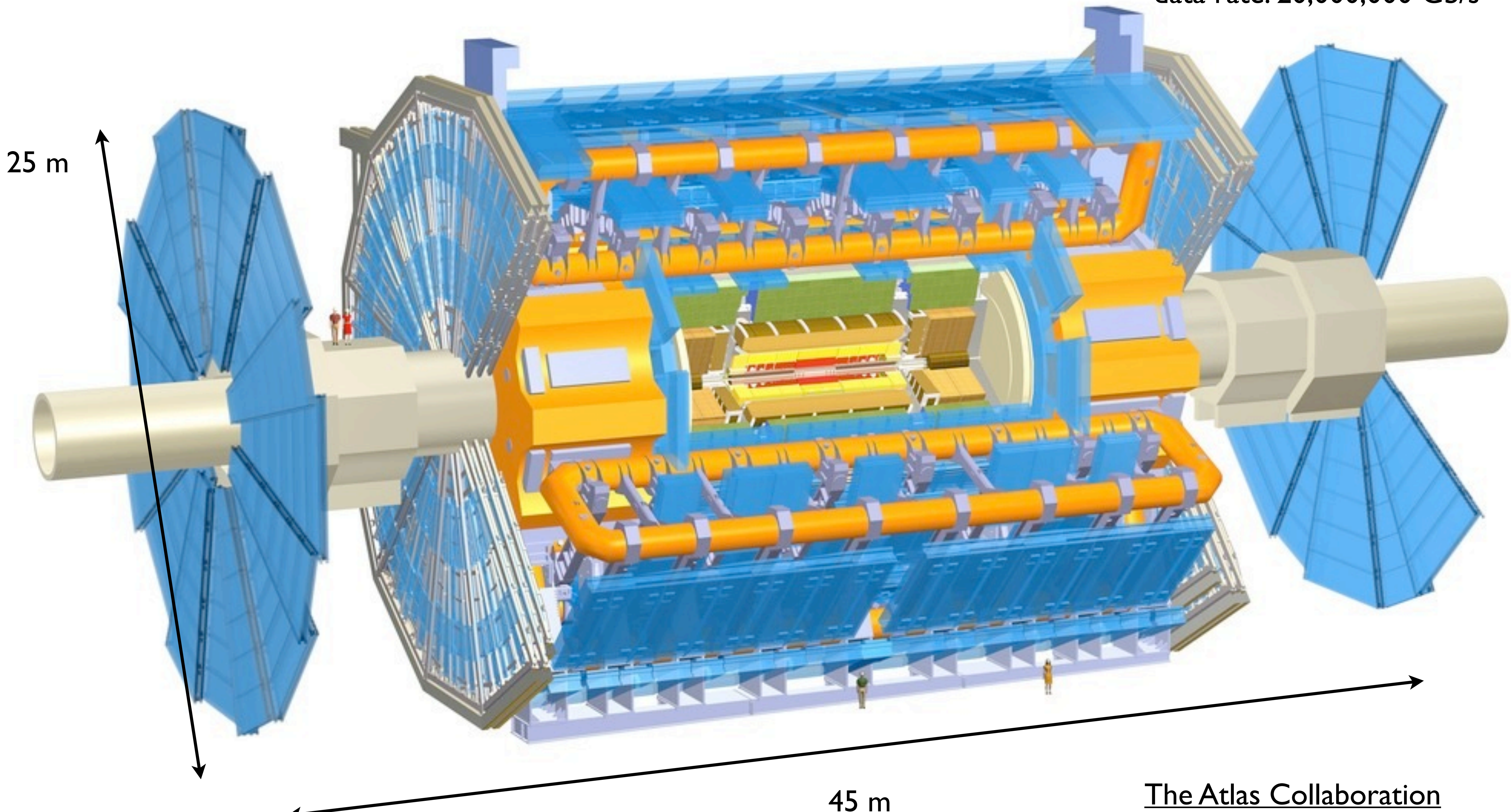
To measure the structure size x use wavelengths of similar size
 - the probing scale

Don't use a finger to probe the structure of a sand grain!

Shorter wavelengths = higher energy
$\Rightarrow$ need more energetic colliders!

The ATLAS experiment at the LHC

<u>The Atlas Experiment</u>
7000 tonnes
Mass of the Eiffel Tower
Half the size of Notre Dame
data rate: 20,000,000 Gb/s



<u>The Atlas Collaboration</u>
3500 physicists
174 universities
38 countries

Particle Signatures

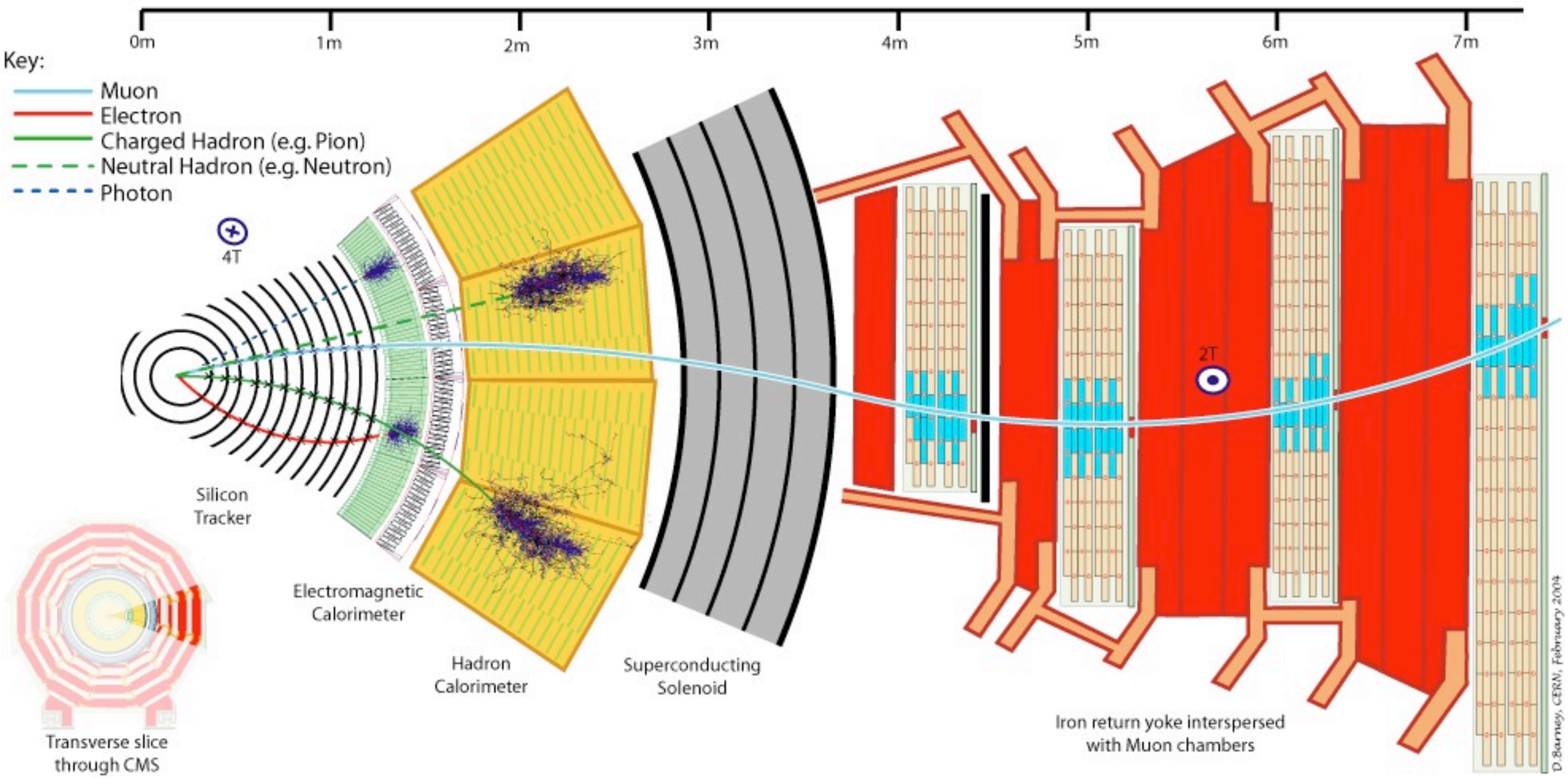
Measuring cross-section of a process requires recognising event properties:

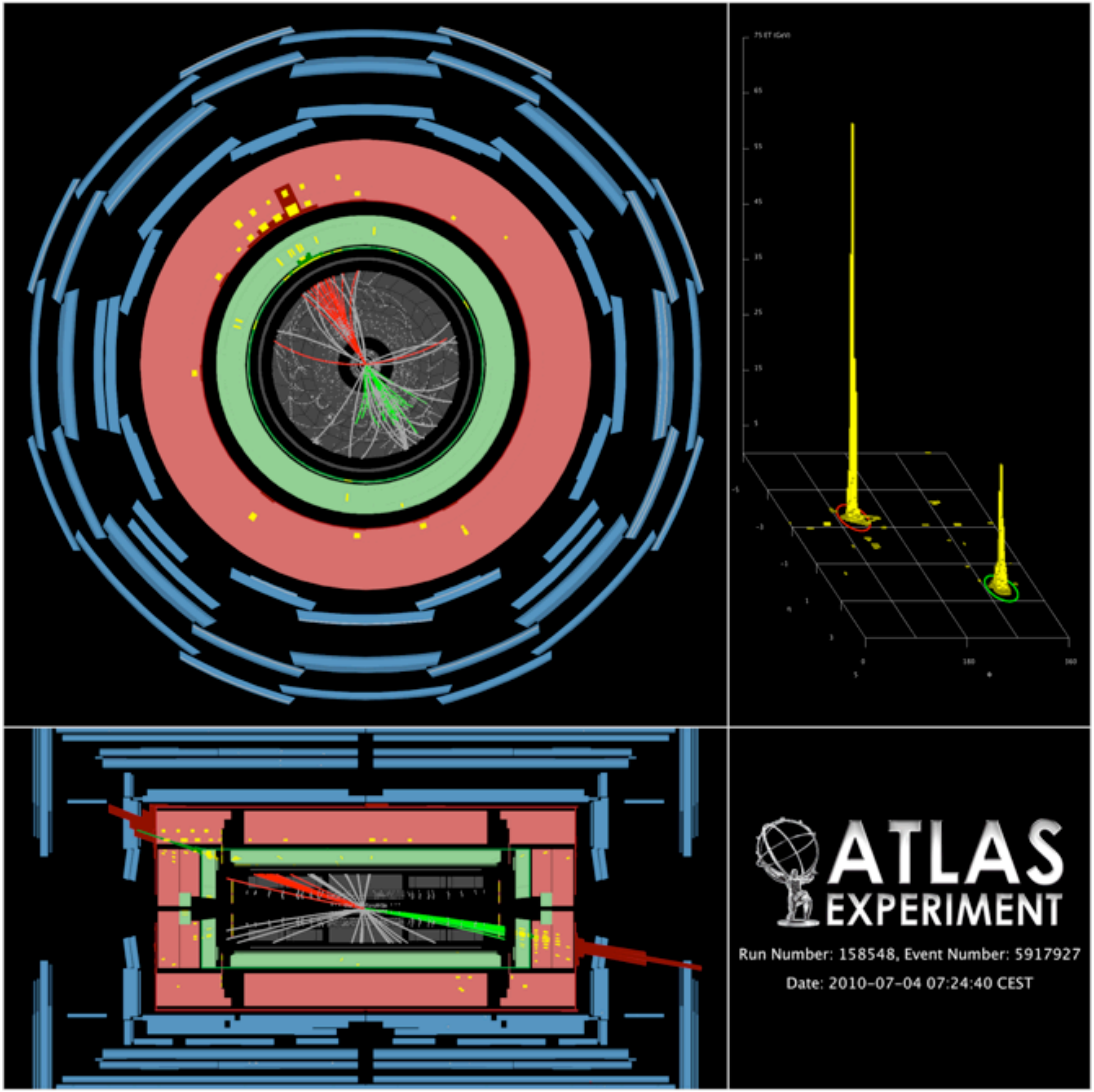
Electromagnetic energy with a charged track	e+ or e-
Electromagnetic energy without track	photon
collimated 'jet' of particles	gluon/quark induced jet
penetrating charged track	μ+ or μ-
missing transverse energy	ν
missing longitudinal energy	beam remnants
displaced secondary vertex	in-flight decay of 'long lived' particle

Look at the event topology...

Particle Signatures

Large experiments needed to measure outgoing particles from collisions
Experiment consists of layered detectors each sensitive to different types of particle
Look for signatures of particle types

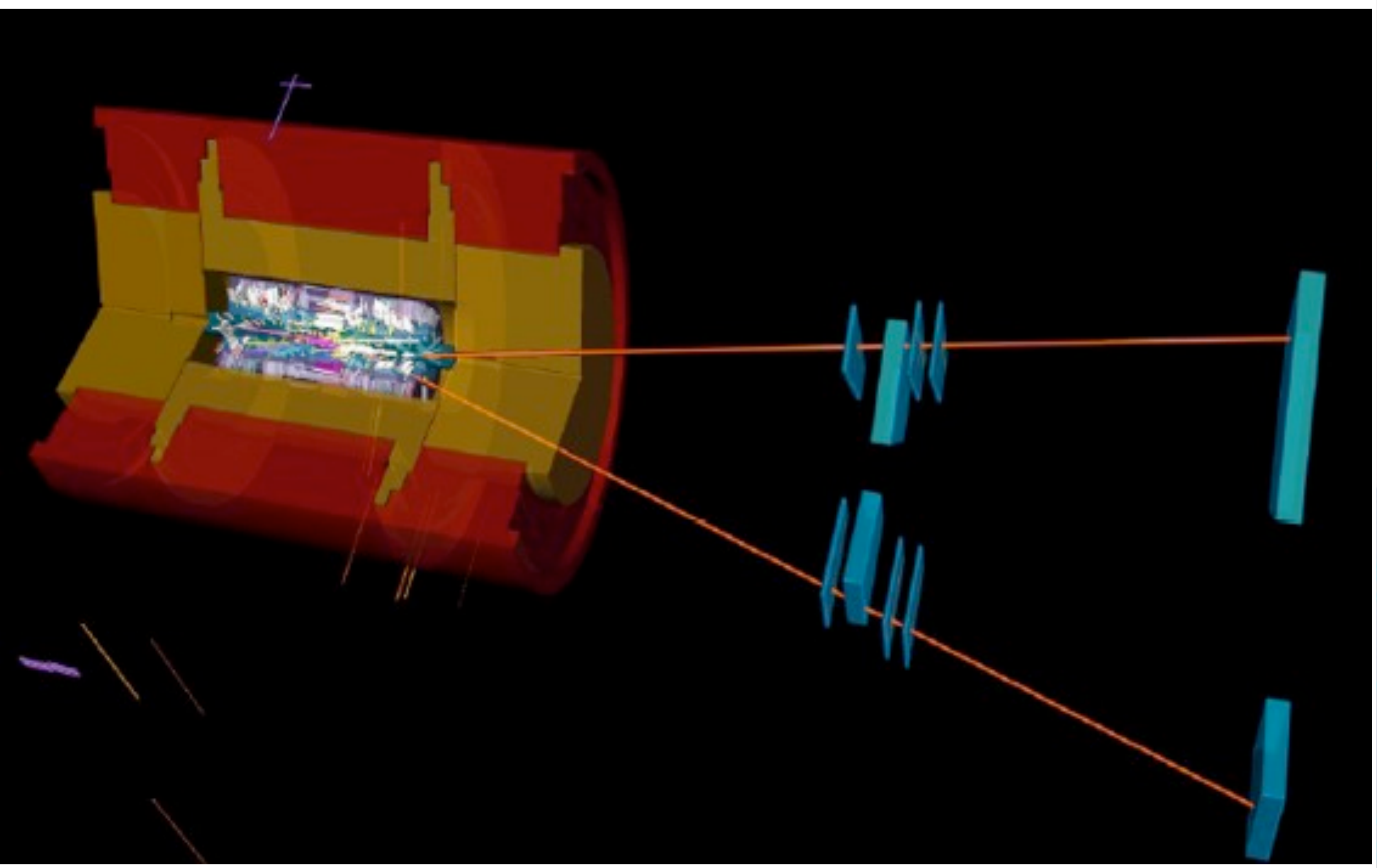


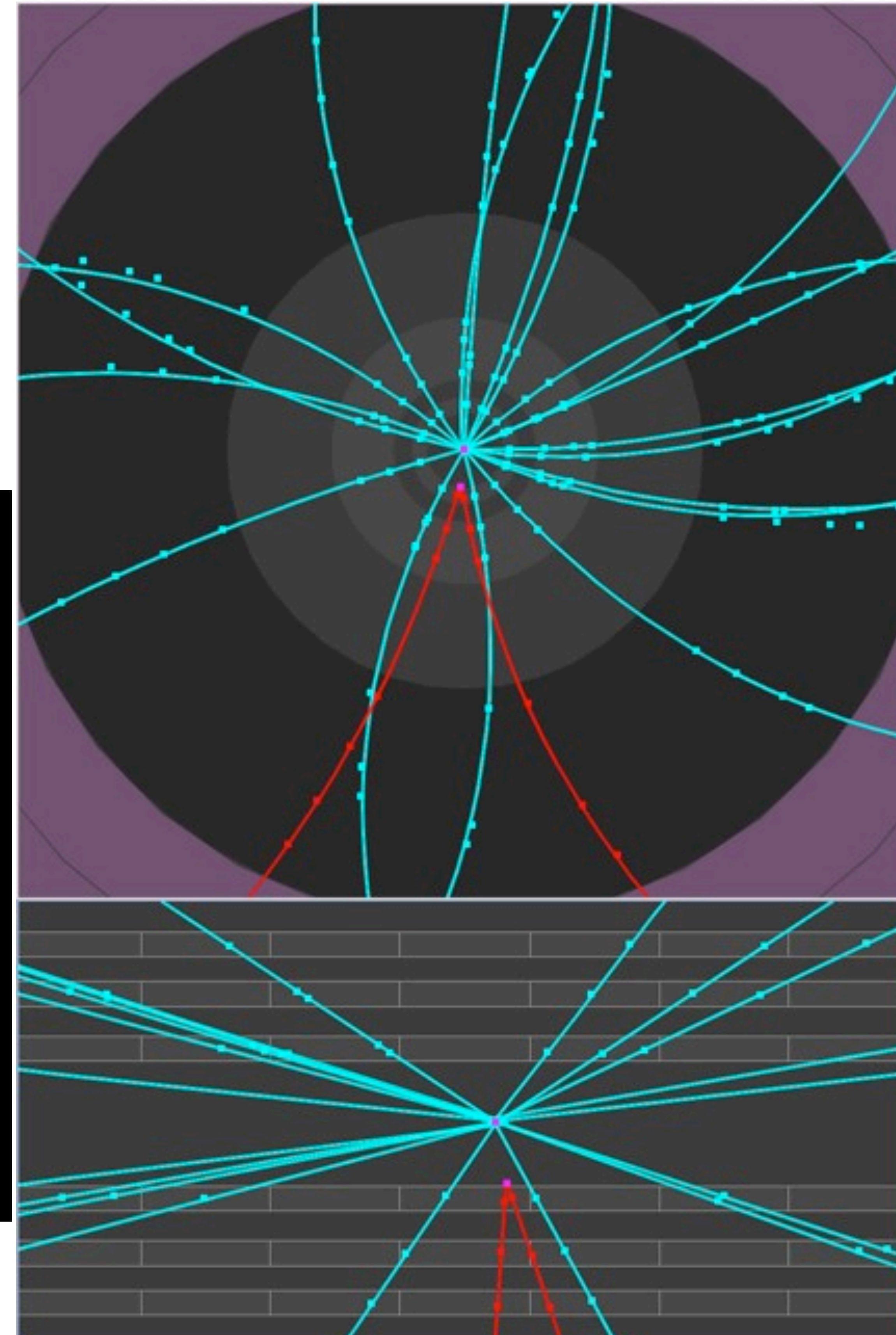


2 jets of particles:
quarks / gluons

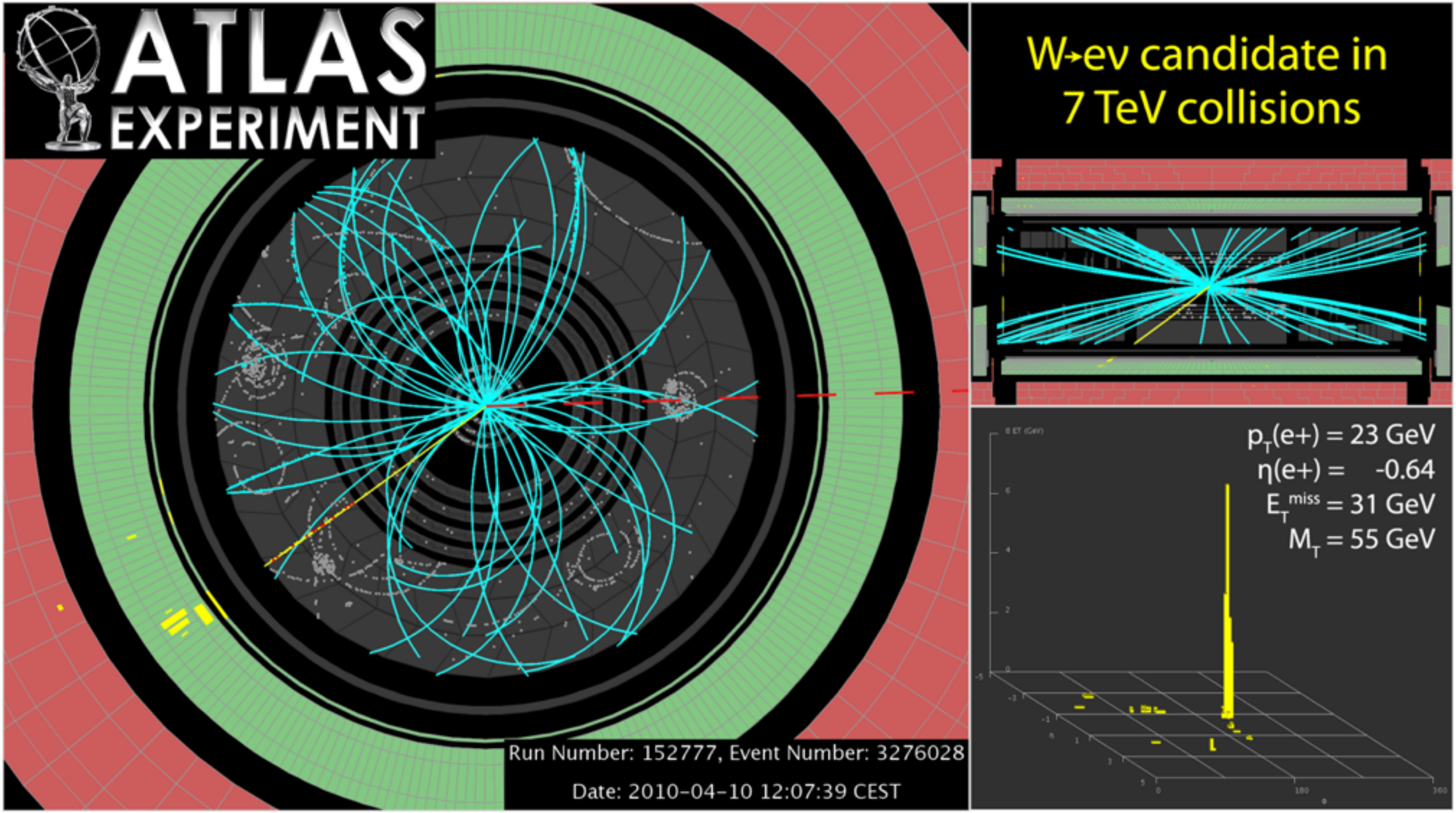
LHC Collision Events

two penetrating particles
opposite charge





LHC Collision Events



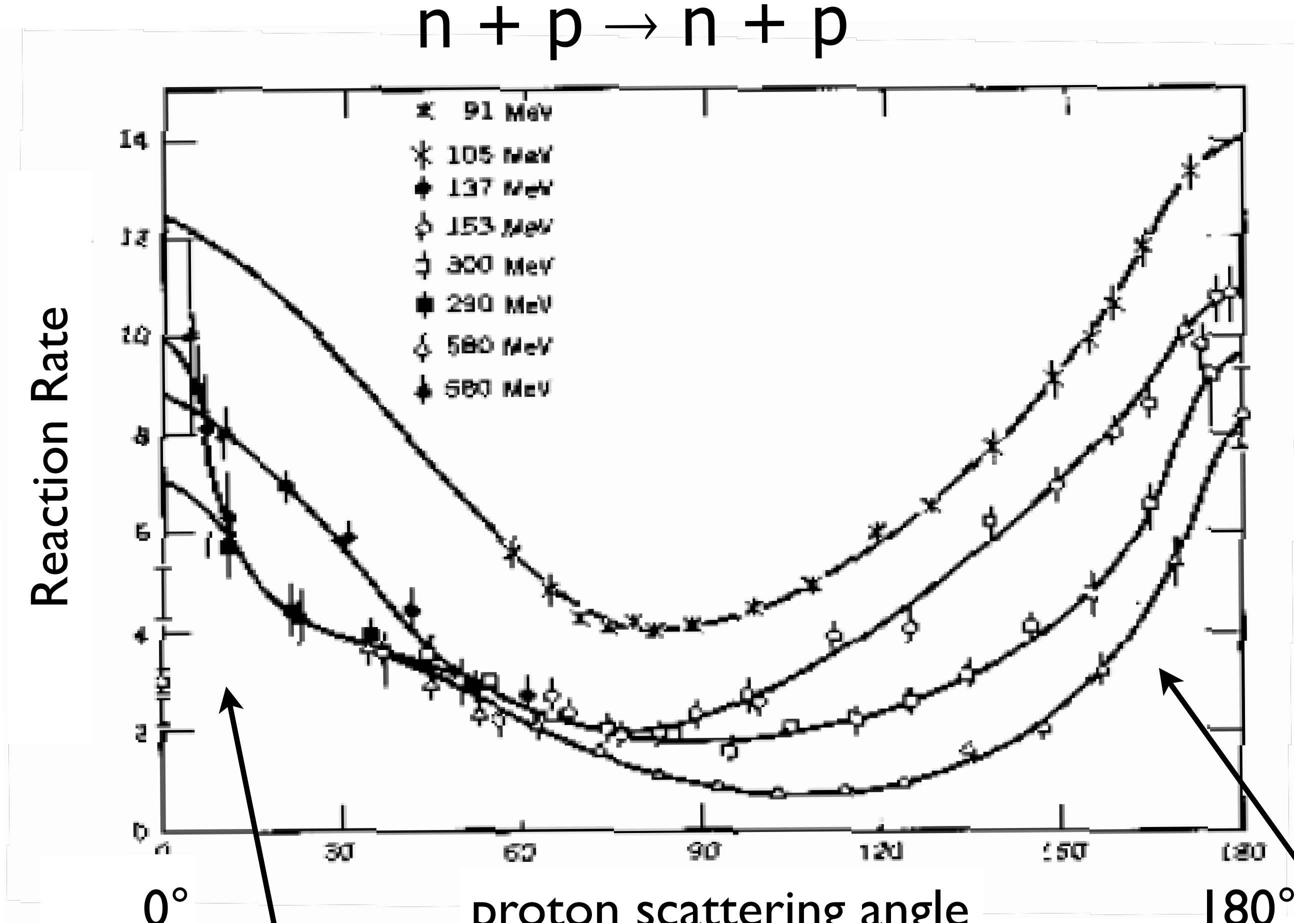
Electromagnetic energy with associated particle track
Missing energy in transverse direction

What’s going on in these collisions?

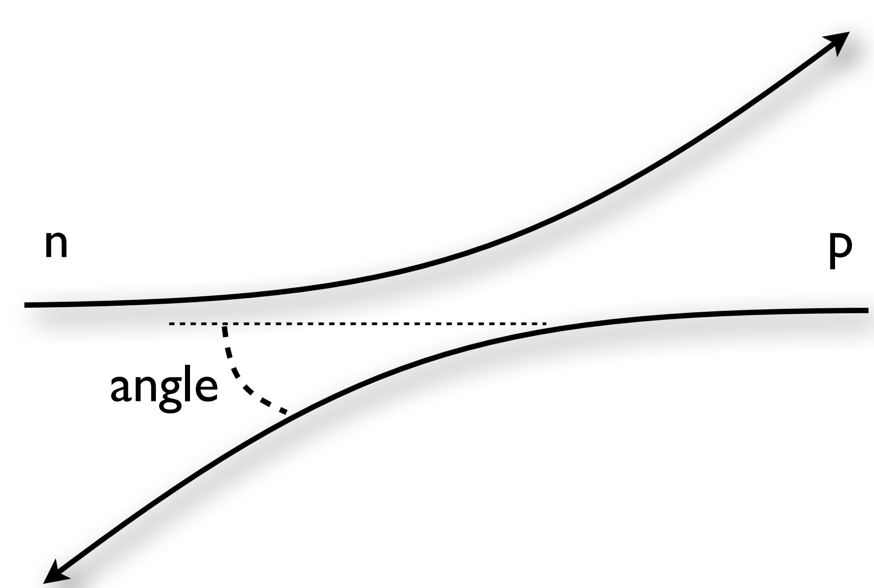
Neutron - Proton Scattering

We measure reaction rates!
Rate at which particles are produced versus energy, momentum, angle...
Called a cross section
related to the probability of a specific reaction occurring

$n + p \rightarrow n + p$



low energy proton / neutron scattering

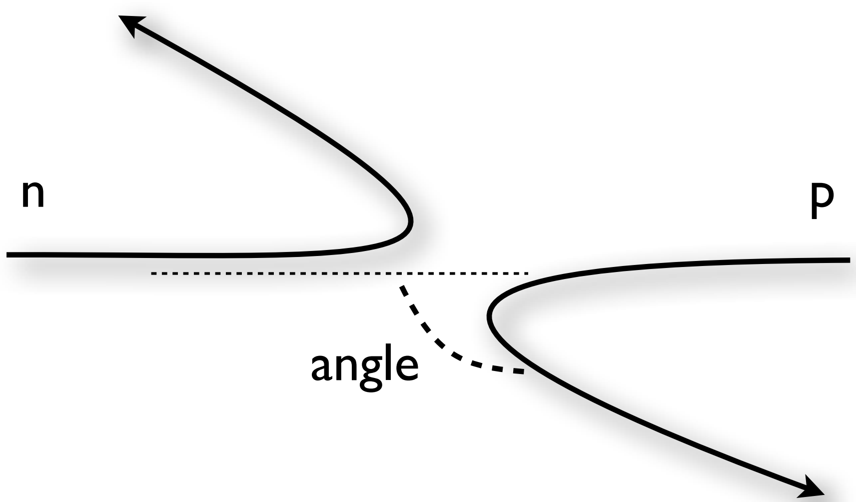


Like Rutherford scattering
Expect largest reaction rate for small angle scattering

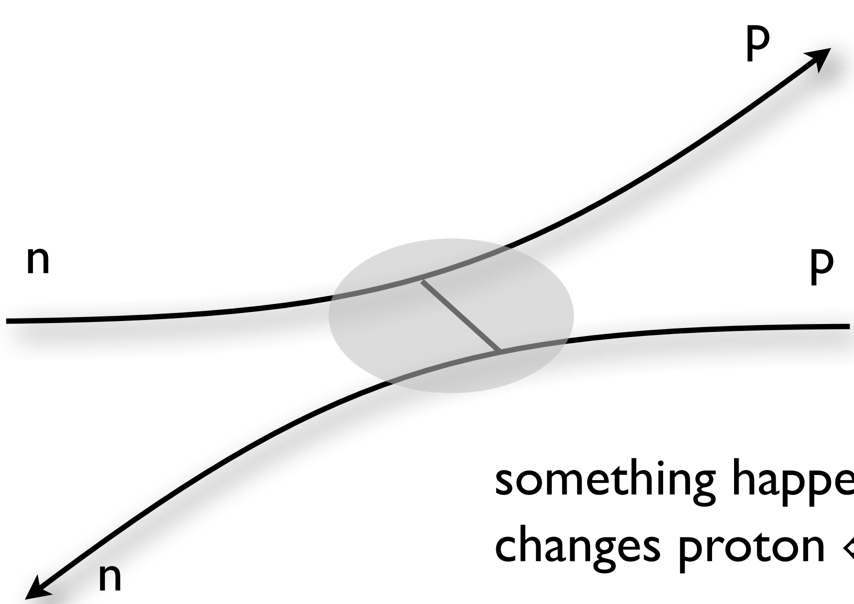
Why is reaction rate equally large here???
Equally likely to have head-on collision as glancing collision??

Neutron - Proton Scattering

large angle scattering



This reaction should occur far more infrequently!

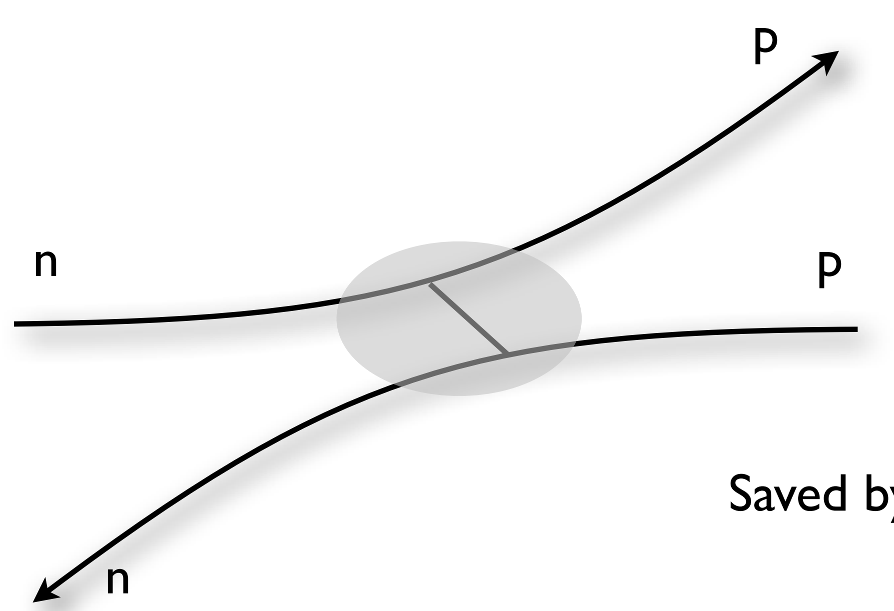


particles exchange identity!

something happens here
changes proton ⇔ neutron

The interaction must:
- exchange momentum
- exchange electric charge

The Exchange Model



An exchange particle is forbidden
violates energy-momentum conservation

Imagine you are moving alongside the proton at equal speed
As far as you can measure the proton is at rest compared to you
Where does it get the energy to emit a particle from?

Saved by the Heisenberg Uncertainty Principle:

$$\Delta E \Delta t > h$$

Small energy ΔE can be 'borrowed' for a time Δt = h / ΔE !

This process is an interaction - it is the expression of a force of nature
Newton: force = rate of change of momentum (F=ma)

What can we predict about the exchange particle?

ΔE is 'used' to produce the particle with mass - what is it?
Nuclear force acts between n and p - has a range of 1 fm
Assume it travels at light speed c - how long does it live for?
$c\Delta t$ = 1 fm & $\Delta E = mc^2$

$$mc^2 \approx \frac{hc}{c\Delta t}$$

So m = 200 MeV/c^2

1/5th of proton mass
400 times electron mass

Our bizarre model predicts new particle:
- mass ~ 200 MeV
- charge = +1
- responsible for force of attraction inside nucleus

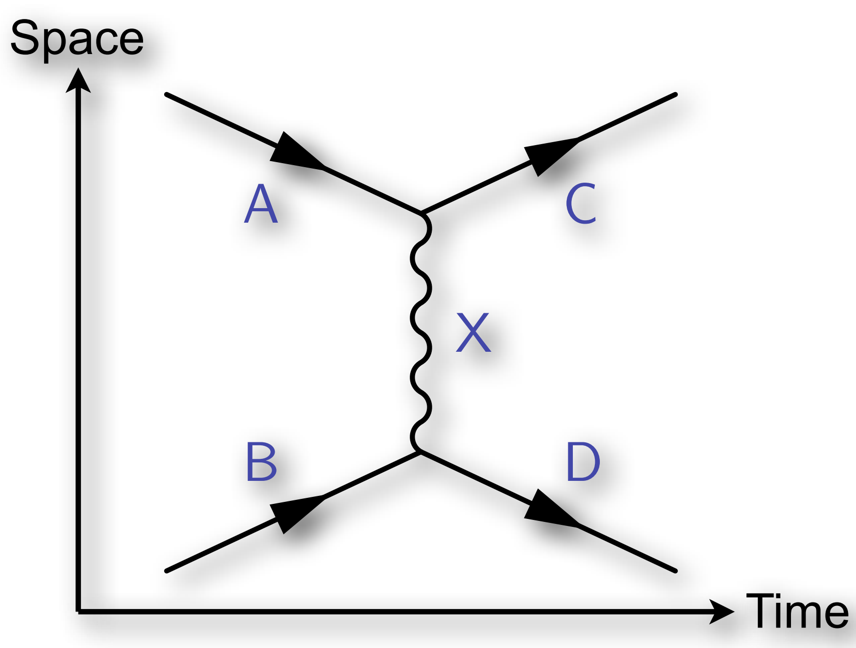
Predicted by Hideki Yukawa in 1934
Also showed that electrostatic force can be described by a massless particle particle exchange
Both particles have spin = 0 or 1
⇒ the photon!

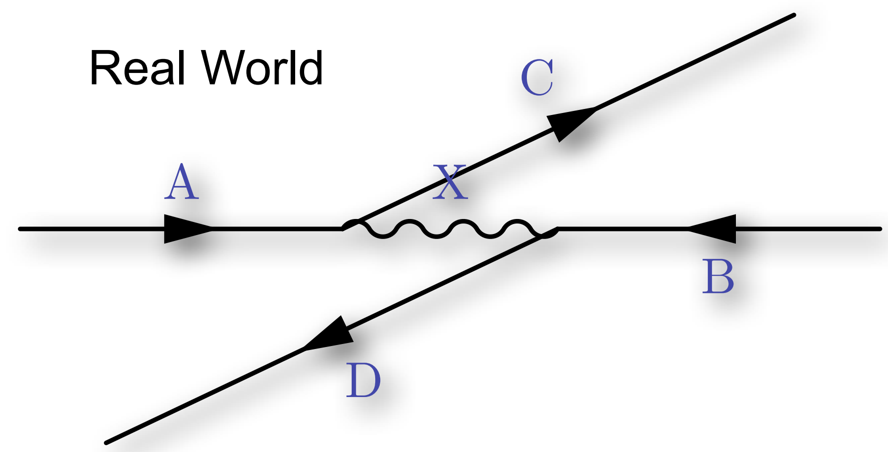
1947 - three particles discovered: the pion

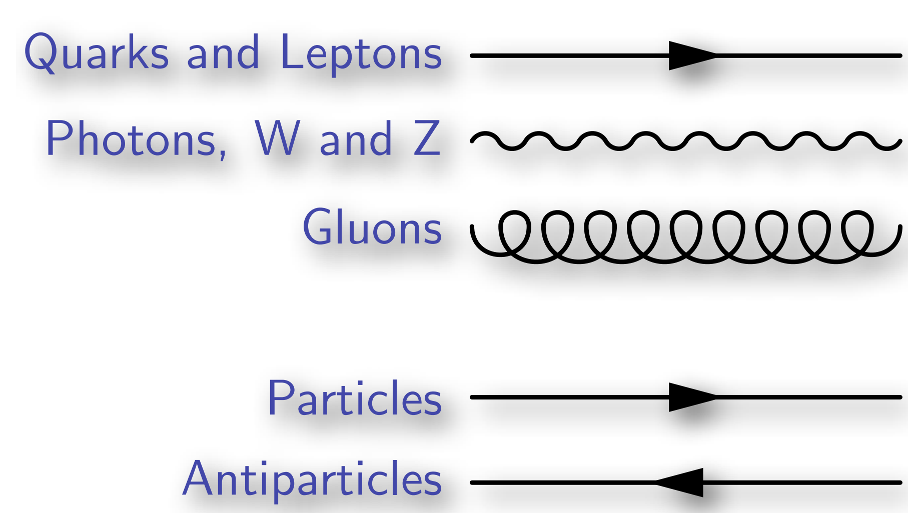
particle	charge	mass
π^0	0	135.0
π^+	+1	139.6
π^-	-1	139.6

The exchange model seems to work!
Extra two particles describe interactions of nn and pp scattering

Modern quantum field theory: fundamental interactions visualised as Feynman diagrams



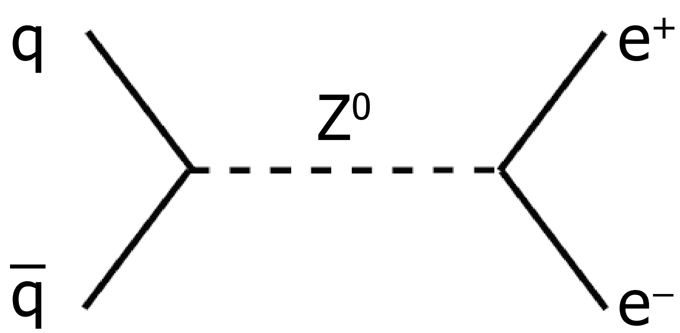




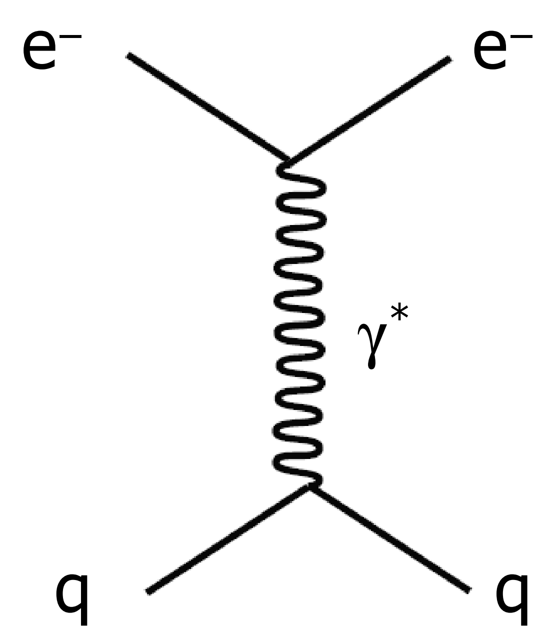
Feynman Diagrams

Examples of Feynman diagrams

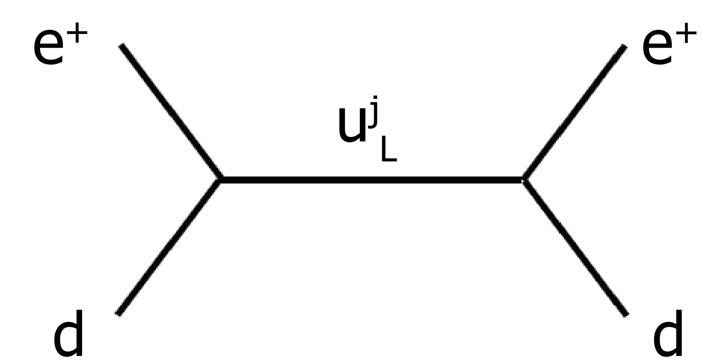
Particle exchange is the manifestation of a force!



$$q\bar{q} \rightarrow e^{+}e^{-}$$



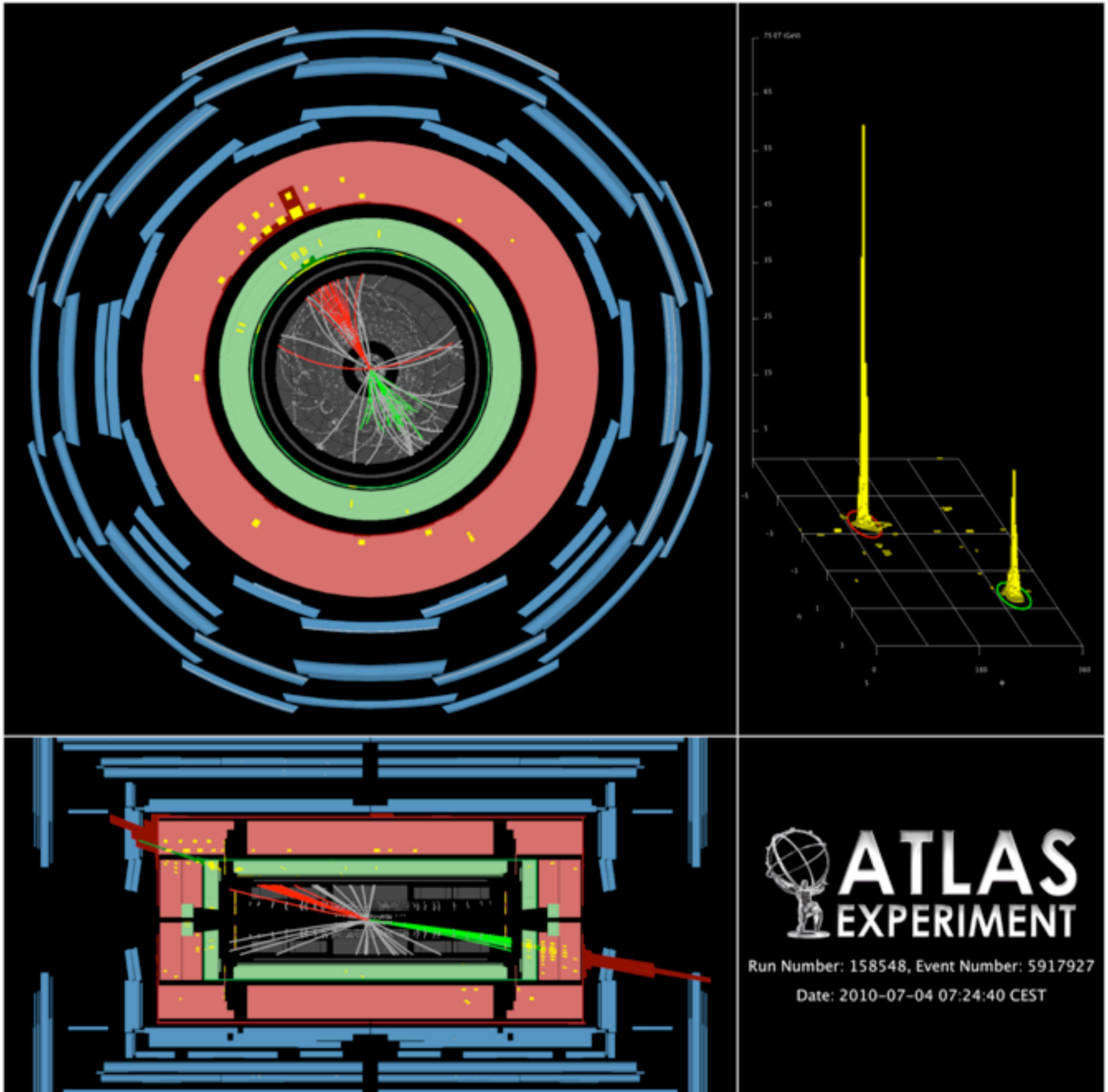
$$eq \rightarrow eq$$



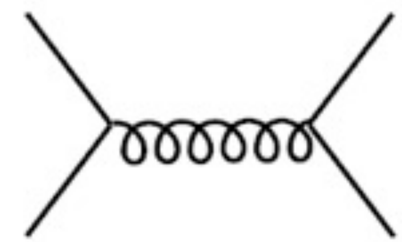
$$e^{+}d \rightarrow e^{+}d$$

Feynman diagrams have an intuitive simplicity
They are more than just aids to visualisation
They represent precise mathematical equations
They allow quantitative predictions for reaction rates!

LHC Collision Events



A 'di-jet' event at high energy



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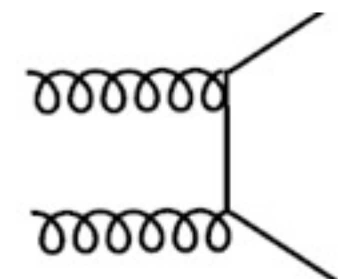

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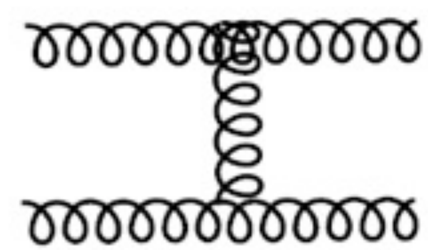
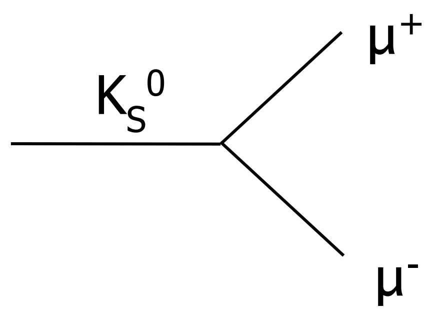
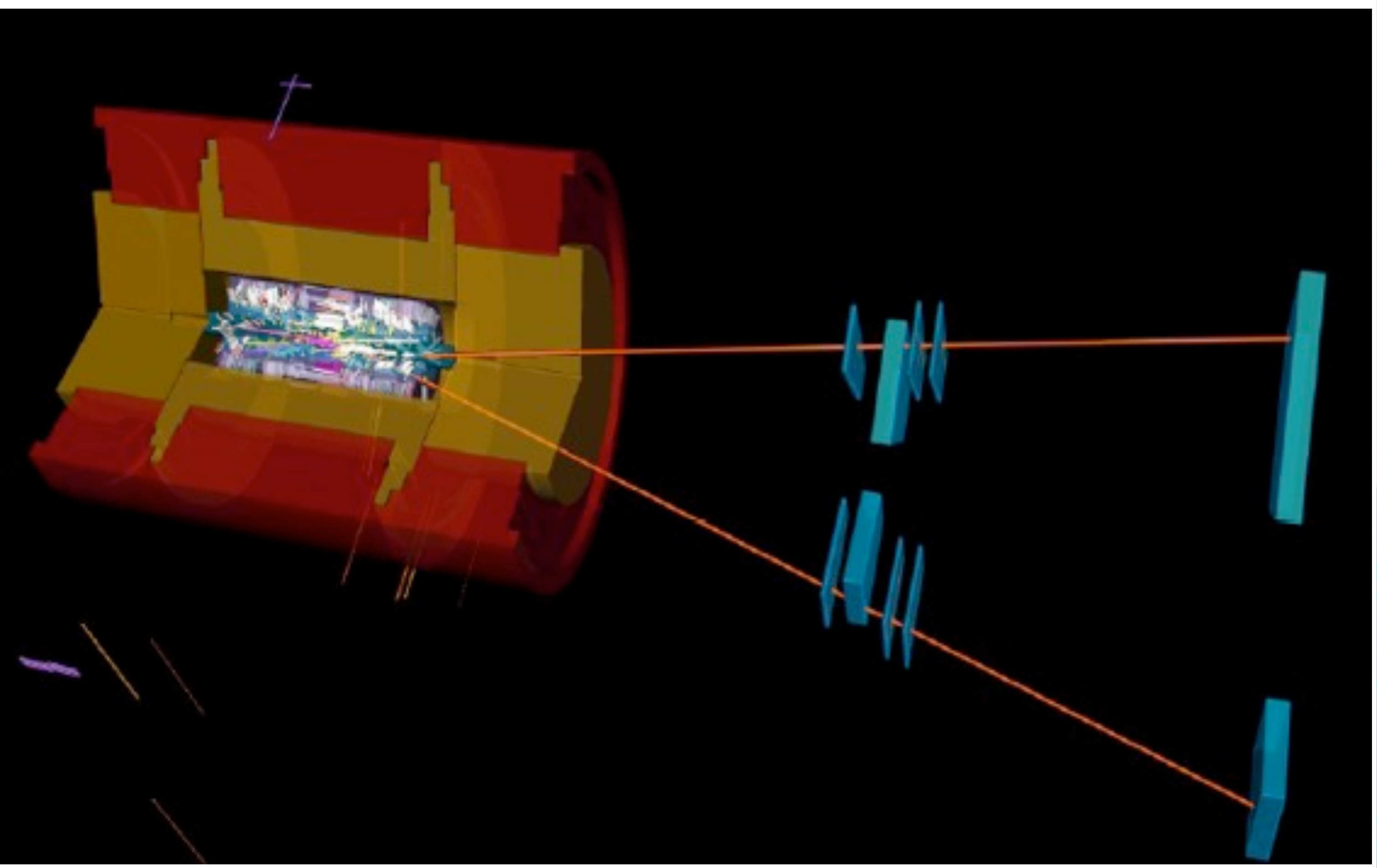
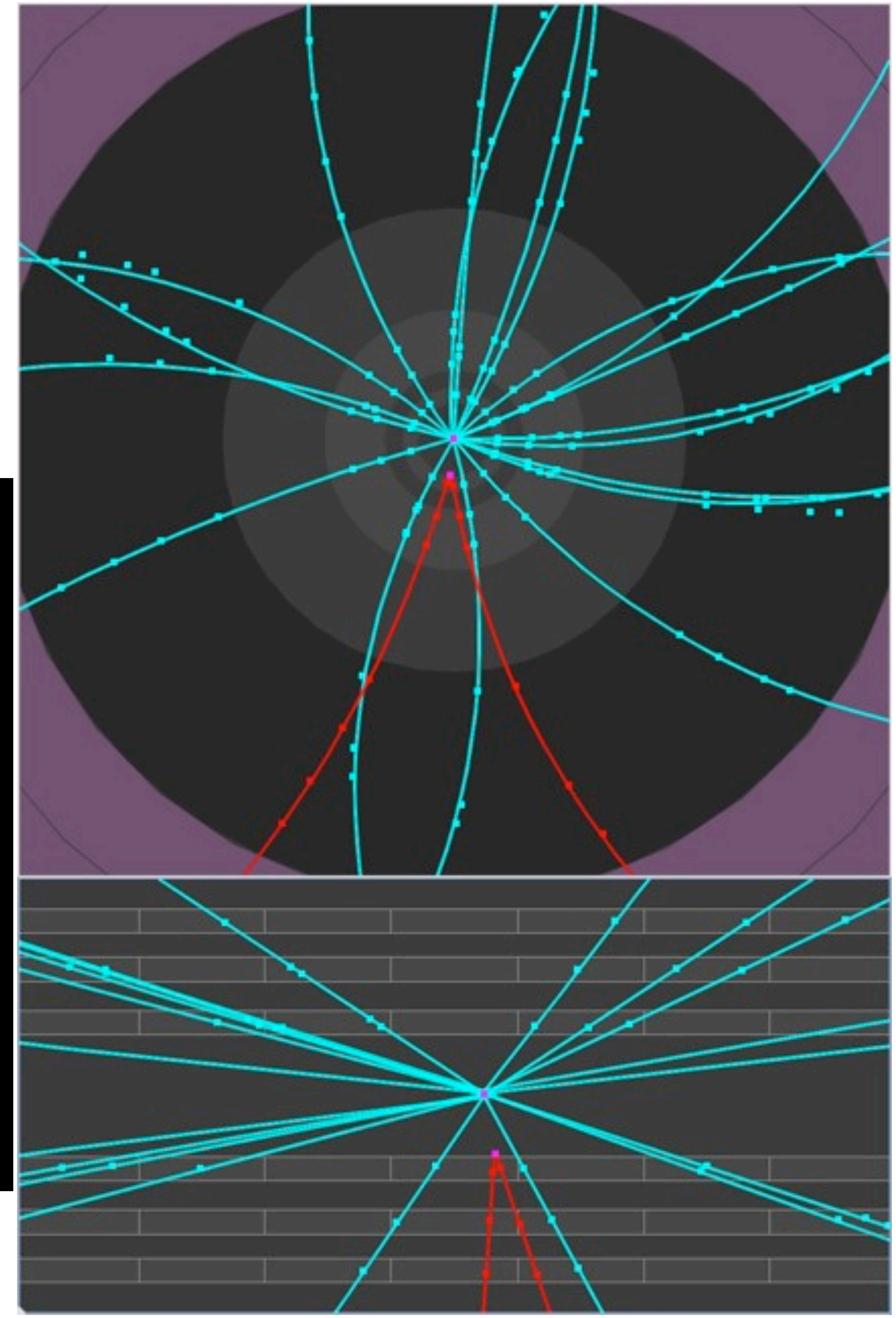


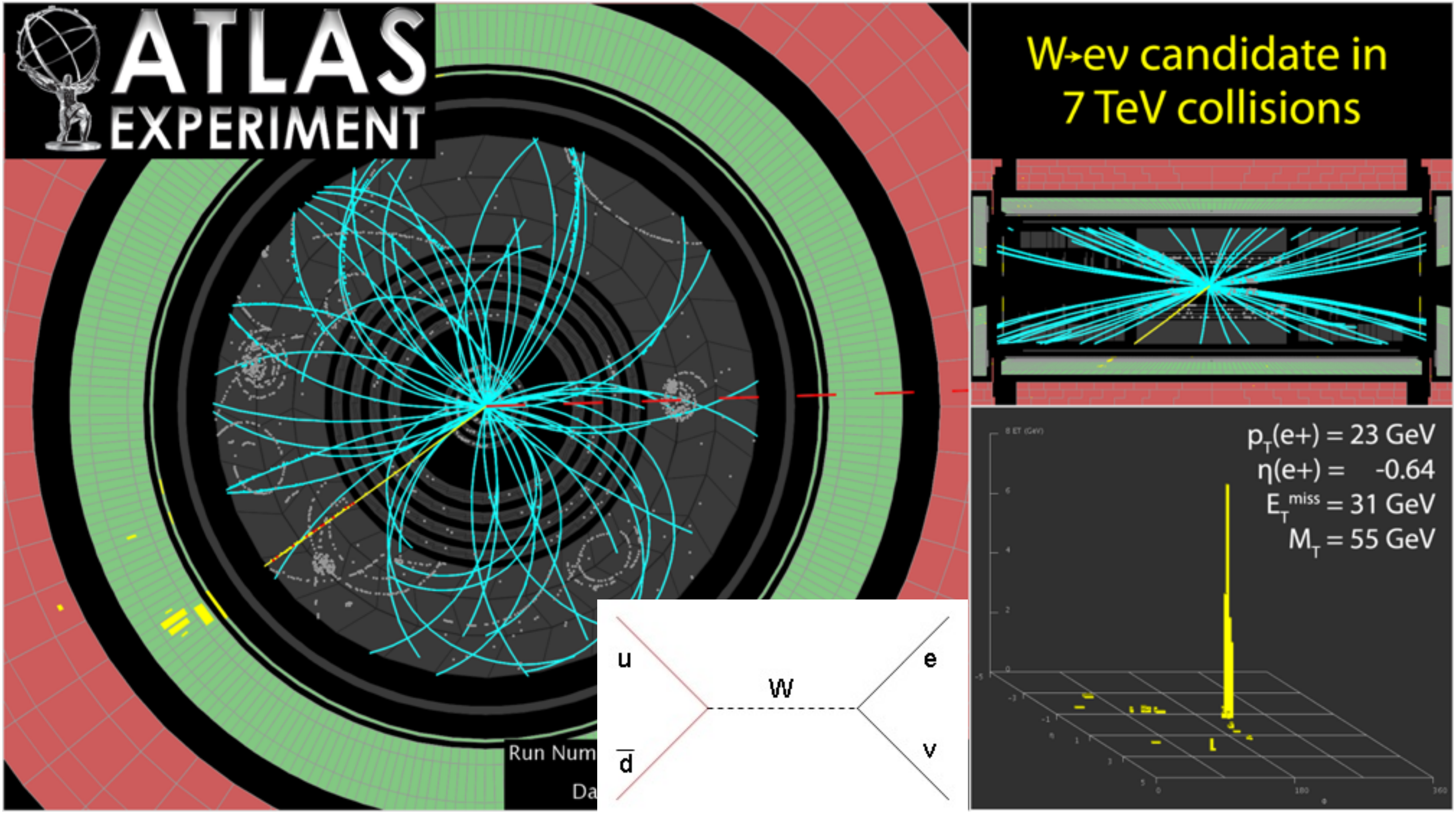
Decay of a long-lived composite particle

- Two oppositely curved tracks
- Penetrating tracks
- Displaced secondary vertex









Production and decay of a W boson particle (carrier of the weak force)

Higher energy → probing particle interactions further back in time
millionths of a second after the big bang

Forces of nature start to behave in similar ways
Consider them as manifestations of a single unified high energy force